Closed-loop transfer function

A **closed-loop transfer function** in control theory is a mathematical expression (algorithm) describing the net result of the effects of a closed (feedback) loop on the input signal to the circuits enclosed by the loop.

1 Overview

The closed-loop transfer function is measured at the output. The output signal waveform can be calculated from the closed-loop transfer function and the input signal waveform.
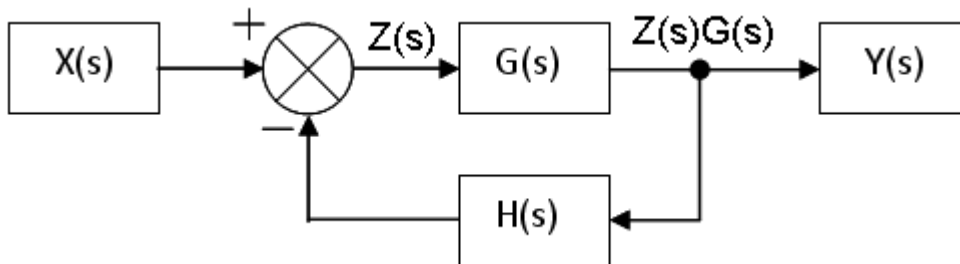
An example of a closed-loop transfer function is shown below:

The summing node and the $G(s)$ and $H(s)$ blocks can all be combined into one block, which would have the following transfer function:

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

2 Derivation

We define an intermediate signal Z shown as follows:

Using this figure we write:

$$Y(s) = Z(s)G(s)$$

$$Z(s) = X(s) - Y(s)H(s)$$

$$X(s) = Z(s) + Y(s)H(s)$$

$$X(s) = Z(s) + Z(s)G(s)H(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{Z(s)G(s)}{Z(s) + Z(s)G(s)H(s)}$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

3 See also

- Federal Standard 1037C

- Open-loop controller

4 References

- This article incorporates public domain material from the General Services Administration document "Federal Standard 1037C".

5   Text and image sources, contributors, and licenses

5.1   Text

- **Closed-loop transfer function** *Source:* https://en.wikipedia.org/wiki/Closed-loop_transfer_function?oldid=656315939 *Contributors:* Andre Engels, Michael Hardy, Merovingian, YUL89YYZ, Mdd, Xhin, Skorkmaz, Jamsignal, Bhny, Grubber, SmackBot, Chendy, Eastlaw, Crunchy Numbers, EagleFan, David Eppstein, Enigmaman, Jjurski, Trschell, The Thing That Should Not Be, Dja25, Alexbot, Legobot, Yobot, Fraggle81, Obersachsebot, BenzolBot, MastiBot, Belchman, ItsZippy, DARTH SIDIOUS 2, Wayne Slam, Rezabot, Dremagon, MusikAnimal and Anonymous: 11

5.2   Images

- **File:Arithmetic_symbols.svg** *Source:* https://upload.wikimedia.org/wikipedia/commons/a/a3/Arithmetic_symbols.svg *License:* Public domain *Contributors:* Own work *Original artist:* This vector image was created with Inkscape by Elembis, and then manually replaced.
- **File:Closed_Loop_Block_Deriv.png** *Source:* https://upload.wikimedia.org/wikipedia/commons/b/b8/Closed_Loop_Block_Deriv.png *License:* CC0 *Contributors:* http://en.wikipedia.org/wiki/File:Closed_Loop_Block_Deriv.png *Original artist:* en:User:Skorkmaz

5.3   Content license

- Creative Commons Attribution-Share Alike 3.0