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(Re)Constructing Mathematical Ontologies and Epistemologies in an Era of Transition

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RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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PROSPECTIVE MIDDLE SCHOOL TEACHERS' EXPERIENCES AND CONCEPTIONS OF MATHEMATICS TEACHING AND LEARNING: A MIXED-METHODS STUDY

Kadian M. Callahan
Kennesaw State University
kmcallahan@kennesaw.edu

The aim of this study was to examine the nature of prospective middle school teachers' (PMSTs') learning experiences during an undergraduate mathematics content course and relationships to their conceptions of mathematics teaching and learning. PMSTs identification of engaging in various types of active-learning opportunities that were valuable for learning mathematics suggests that their conceptions of the teaching and learning process may be changing as a result of engaging in those experiences.

Active learning involves talking, listening, writing, reading, and reflecting (Meyers & Jones, 1993) and is infused throughout the National Council of Teachers of Mathematics' ([NCTM's], 2000) Process Standards in *Principles and Standards for School Mathematics* and throughout the Standards for Mathematical Practice in the *Common Core State Standards for Mathematics* (Common Core State Standards Initiative's ([CCSSI], 2010). Talking – communicating one's thinking and constructing viable mathematical arguments – clarifies students' thinking because it requires them to organize and structure their thoughts in a meaningful way so that they can communicate clearly (Meyers & Jones, 1993). Active listening – evaluating others' mathematical thinking – involves listening attentively to others' comments and ideas, considering others' perspectives relative to one's own, and asking questions when something is unclear or confusing (Cobb, 2000; Yackel & Cobb, 1996). Writing – using mathematical language and symbols to communicate one's ideas in written form – encourages students to engage in analytic and synthetic activities that help them to expand, modify, and create mental structures (Hobson, 1996). Reading – making sense of a problem situation and relevant information – engages students in scanning, identifying, sorting, and prioritizing information (Meyers & Jones, 1993). Reflecting – considering connections between and among mathematical ideas and to real-life contexts – creates an opportunity to gather one's thoughts, mull over, sort, out, try to understand, and incorporate new information (Meyers & Jones, 1993). Research shows that actively engaging students in learning mathematics has a positive influence on their problem-solving abilities, reasoning skills, and thinking processes (Davis & Maher, 1997; Doerr & Tripp, 1999; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, et al., 1997; Pirie & Kieren, 1994; Martin, McCrone, Bower, & Dindyal,

2005); exposes them to different ways of thinking about and solving problems (White, 2003); and improves their ability to generalize mathematical relationships (Ellis, 2011; Lannin, 2003).

Research on practicing teachers' implementation of standards-based instruction suggests that their conceptions of the teaching and learning process are framed by their own learning experiences and the value they place on that learning (Knuth, 2002; Stylianou, 2010). Hence, the way in which teachers emphasize different aspects of mathematics instruction – such as actively engaging students in the learning of mathematics – relies heavily on their conceptions of what is important and useful to support mathematics learning. One way to influence teachers' conceptions of mathematics teaching and learning may be to engage them in active learning during mathematics content courses taken during teacher preparation programs (CBMS, 2001; National Mathematics Advisory Panel, 2008; National Research Council, 2001).

This study sought to examine the nature of prospective middle school teachers' (PMSTs') learning experiences during an undergraduate mathematics content course specifically designed for this population and relationships to their conceptions of mathematics teaching and learning. The guiding research questions were:

1. What types of opportunities did PMSTs have to actively engage in learning mathematics?
2. How did opportunities to actively engage in learning mathematics relate to PMSTs' sense of how to foster mathematics learning?

Theoretical Perspective

The notion of fostering students' learning through discourse is grounded in sociocultural theories of learning based primarily on the work of Lev Vygotsky, who emphasized that social interactions are an essential element of cognitive development. Vygotsky argued that there is fluidity between self and others, and cognitive exchanges occur at this boundary. These exchanges mitigate the process through which knowledge development occurs, subsequently influencing movement of a student beyond what he might have been able to do on his own (Rogoff, 1999; Wertsch, 1985). According to Cobb (2000), classroom discourse enhances the learning process when it is used as a tool to promote the exchange of ideas in mathematics classrooms. "Additional learning opportunities arise when children attempt to make sense of explanations given by others, to compare others' solutions to their own, and to make judgments about similarities and differences" (Yackel and Cobb, 1996, p. 466). Thus, by discussing mathematical ideas with others, students begin to reason about and make connections among and between those ideas.

Methods and Methodology

This study was conducted at a four-year public university in the southeastern United States. The 22 participants (14 female; 8 male) were PMSTs enrolled in a mathematics content course for middle and secondary teachers during the spring 2011 semester. The emphasis of the course was on involving the PMSTs in an in-depth study of the middle school content strands of number and operations, algebra and functions, and geometry and measurement while immersing them in the processes of mathematical inquiry (NCTM, 2000; CCSSI, 2010). Throughout the course PMSTs were asked to reason about and make connections between and among representations of mathematical ideas across these three content strands. PMSTs were also encouraged and expected to regularly engage in discussions with their peers as they shared their thinking about problems and solution strategies, reasoned about and made connections between mathematical ideas, made and evaluated conjectures, and developed and revised mathematical arguments.

This study employed a convergent parallel mixed methods research design that involved separate quantitative and qualitative data collection and analyses and integrated results (Creswell & Plano Clark, 2011). The reason for collecting both quantitative and qualitative data was to corroborate results arising from different sources to bring greater insight into the types of learning opportunities that PMSTs engaged in (quantitative component) and how they felt those experiences influenced the learning of mathematics (qualitative component). The quantitative data were gathered at the end of the semester from the *Mathematics Active Learning Experiences Survey* (Callahan, 2006). PMSTs were asked to indicate the frequency which they engaged in each of 13 active-learning opportunities by selecting "Not at All," "About Once or Twice a Month," "About Once Every Week," or "Almost Every Lesson." During the semester that this data was collected the course met once a week so the last two options were collapsed into one category ("Almost Every Lesson") during analysis. Survey data were analyzed using descriptive statistics to explore the context and frequency of PMSTs' participation in active-learning opportunities during the course. The qualitative data were collected from an end-of-semester writing assignment that asked each PMST to identify five aspects of mathematics content or mathematics teaching that they understood more deeply or differently than they did before taking the course. Writing assignment data were analyzed using the constant comparison method (Strauss, 1987). Patterns and themes were identified, merged, discarded, and revised as the researcher compared and contrasted PMSTs' written responses.

Results

18 PMSTs completed the *Mathematics Active Learning Experiences Survey*. All 18 (100%) indicated that they engaged in some type of active-learning experience during almost every lesson. As shown in Table 1, more than 75% of the PMSTs noted that they engaged in 10 out of the 13 active-learning opportunities during "Almost Every Lesson". Notably, seven (7) of the 10 most frequently cited opportunities involved students working together. These results suggest that not only did the PMSTs had regular opportunities to actively engage in learning mathematics in ways that are espoused by the standards for school mathematics, but that exchanging ideas with their peers was central to these experiences.

Table 1

Reported Active-Learning Experiences Engaged in "Almost Every Lesson"

	Freq (n=18)	Percent
"I listened to and evaluated other students' ideas, solutions, or points of view."	17	94.44
"I was challenged to defend, extend, clarify, or explain how I derived my answers or ideas."	15	83.33
"I was expected to 'investigate' or 'discover' mathematical principles and ideas."	16	88.89
"I worked with other students to explore new ideas/concepts through problem examples."	17	94.44
"I shared strategies with other students for approaching or solving a problem."	16	88.89
"I justified my reasoning in a problem or steps in a proof."	14	77.78
"I discussed connections between mathematical ideas/concepts with other students."	16	88.89
"I worked with other students to evaluate or construct proofs or make conjectures/propositions."	15	83.33
"When students were working together, we were encouraged to admit confusion and ask questions."	15	83.33
"I taught a particular mathematical idea to the class."	7	38.89
"I directed questions to other students about mathematical ideas/concepts."	10	44.44
"I put individual or group work on the board for classmates to examine or comment on."	11	33.33
"I worked in groups with other students on projects to be turned in for a grade or extra credit"	14	77.78

Findings from survey data were supported by data from the writing assignment. 20 PMSTs completed the writing assignment. In their responses, PMSTs consistently spoke of how helpful it

was to reason about the mathematics by engaging in discussions with their peers. Noelle (all names are pseudonyms) talked about working with her peers to evaluate conjectures.

> We came up with our own conjectures about rectangular arrays. … We were able to evaluate each conjecture by adding or taking away words that made [them] true or false. By stating in our own words if the conjectures were true or false we formed arguments and were able to find ways to prove the conjectures we made while making new ones.

The PMSTs also discussed the connections they made within and across mathematics strands by working with peers to consider different representations of the same idea. Gabriela described how her understanding of even and odd numbers changed as a result of the class being pressed to make connections between numeric, algebraic, and geometric representations.

> Tilo's model [a geometric representation found in Connected Mathematics (Lappan, Fey, Fitzgerald, Friel, Phillips, 2006)] physically showed me how to represent even and odd numbers. With class discussion and group work, I learned how to take Tilo's model and develop an algebraic formula for expressing even and odd numbers. Before this class I hadn't understood the algebraic formulas of expressing even and odd numbers. With the use of Tilo's model, I was able to fully understand how and why the formulas work.

Other PMSTs, like Ellis, expressed learning new things about the nature of mathematics teaching and learning. In his reflection he said,

> I see now that one of the best ways to bring people to a better understanding of math concepts is to let them discover it themselves. I don't think any of the math concepts we covered were exactly new to anyone, but it was interesting that approaches like small group work and discussion really do facilitate a better learning process. Instead of just lecturing, we were encouraged to "rediscover the wheel," and it really does work.

> The PMSTs' sentiments shared in the writing assignment suggest that their sense of the value of learning mathematics through active-learning experiences may be changing. Furthermore, many of the PMSTs seemed surprised by how much more they learned about connections and representations by working with their peers to make sense of the mathematics for themselves.

Discussion

Given the interactive nature of mathematics teaching and learning, the opportunities that students have to experience standards-based instruction is tied closely to pedagogical practices (e.g.,

Atkins, 1999; Martin, et al., 2005; Pierson, 2008; White, 2003). Since pedagogical practices are heavily influenced by what teachers see as valuable from their own learning experiences (Knuth, 2002; Stylianou, 2010), it is pertinent that the learning opportunities they have in mathematics content courses during their teacher preparation programs help them to see active learning as valuable in their own learning of mathematics. The results of the present study suggest that this group of PMSTs' saw their active-learning experiences as being valuable in strengthening their ability to reason about and make connections between and among mathematical ideas and different representations of those ideas. In addition, they credited changes to their thinking about mathematics teaching and learning to regular interactions with their classmates. Additional research is needed to look more closely at specific types of peer interactions to identify how and to what extent they influence PMSTs' understanding of mathematics content and mathematics teaching. Research is also needed to determine how such experiences during undergraduate study relate to the types of active-learning opportunities teachers' afford to their students in their own classrooms.

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A COMPARISON OF TWO ALTERNATIVE PATHWAY PROGRAMS IN SECONDARY MATHEMATICS TEACHER CERTIFICATION

Brian R. Evans
Pace University, New York
bevans@pace.edu

The purpose of this study was to compare the mathematics content knowledge, attitudes, and efficacy held by teachers in two alternative pathways to mathematics teacher certification: New York City Teaching Fellows (NYCTF) and Teach for America (TFA). Differences were not found in content, attitudes, and efficacy, but learning and teaching journals revealed several differences between NYCTF and TFA teachers. Particularly, social justice in the classroom was mentioned more often by TFA teachers, and NYCTF found classroom management to not be as much an issue as had been expected.

The purpose of this study was to compare the mathematics content knowledge, attitudes toward mathematics, and concepts of efficacy held by teachers in two alternative pathways to mathematics teacher certification: New York City Teaching Fellows (NYCTF) and Teach for America (TFA). Secondly, the purpose was to determine differences in their attitudes toward their own learning and teaching as new mathematics teachers in New York City.

Strong teacher content knowledge is an important factor for teaching mathematics successfully (Ball, Hill, & Bass, 2005). Negative teacher attitudes toward mathematics often lead to avoidance of teaching strong mathematical content and affect students' attitudes and behaviors (Amato, 2004). Poor attitudes toward teaching are directly related to teacher retention issues (Costigan, 2004), and efficacy is an important component for successful teaching since efficacy is a teacher's belief in his or her ability to teach effectively and positively affect student learning outcomes (Bandura, 1986; Enochs, Smith, & Huinker, 2000).

The NYCTF program is an alternative certification program developed in 2000 in conjunction with The New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007). The program goal was to recruit professionals from other fields to supply the large teacher shortages in New York City's public schools with quality teachers. Prior to September 2003, New York State allowed for teachers to obtain temporary teaching licenses to help fill the teacher shortage. Teaching Fellows generally are recruited to teach in high needs schools throughout the city (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2006).

TFA is a non-profit organization formed in 1990 with the intention of sending college graduates to low-income schools to make a difference for the underserved students. Its founder, Wendy Kopp, was herself a new graduate of Princeton University looking to do something more with her

life after graduation (Kopp, 2003). She considered that many recent college graduates at America's top universities would consider teaching low-income students if given the opportunity. The idea was that there should be a teachers' corps that would allow new graduates at top universities with an interest in teaching to quickly begin teaching students in underserved communities.

Theoretical Framework

Aiken (1970) was an early pioneer to examine the relationship between mathematical achievement and attitudes toward mathematics, and showed that attitudes and achievement in mathematics are reciprocal. Ma and Kishor (1997) found a small but positive significant relationship between achievement and attitudes through meta-analysis. This relationship, along with Ball et al.'s (2005) emphasis on the importance of content knowledge for teachers, formed the framework of this study. Ball et al. said, "How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (p. 14). Further, Ball et al. suggested that teachers with high content knowledge could help narrow the achievement gap in urban schools. NYCTF and TFA teachers are placed in high need urban schools in New York City.

Additionally, Bandura's (1986) construct of efficacy theory framed this study's focus on efficacy in NYCTF and TFA teachers. Bandura found that teacher efficacy can be subdivided into a teacher's belief in his or her ability to teach well, and his or her belief to affect student learning outcomes. Teachers who feel that they cannot effectively teach mathematics and affect student learning are more likely to avoid teaching from an inquiry and student-centered approach with real conceptual understanding (Swars, Daane, & Giesen, 2006).

Research Questions

1. What differences existed between NYCTF and TFA mathematical content knowledge, attitudes toward mathematics, and concepts of teaching efficacy both at the beginning and end of a mathematics methods course?

2. What differences existed between NYCTF and TFA attitudes as measured by learning and teaching journals?

Methodology

The study used quantitative and qualitative methods, and participants consisted of 42 NYCTF and 22 TFA teachers at a partnering university, a medium-sized urban university located in New York City. Both NYCTF and TFA teachers took the New York State Content Specialty Test (CST)

in mathematics the summer before they began their program, which is required by the State of New York for teacher certification. Further, both NYCTF and TFA teachers took a mathematics content test at the beginning and end of their mathematics methods course during a semester in which they were teaching in the classrooms. The mathematics content test consisted of 25 free response items ranging from algebra to calculus.

NYCTF and TFA teachers were given a survey instrument that measured attitudes toward mathematics at the beginning and end of the mathematics methods course. The attitudinal questionnaire was adapted from Tapia (1996) and had 39 Likert items. The efficacy instrument was adapted from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) developed by Enochs et al. (2000), and measured concepts of efficacy. The MTEBI is a 21-item five-point Likert scale instrument, and is grounded in the theoretical framework of Bandura's (1986) efficacy theory. The MTEBI contains two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) with 13 and 8 items, respectively. The PMTE specifically measured a teacher's self-concept of his or her ability to effectively teach mathematics. The MTOE specifically measured a teacher's belief in his or her ability to directly affect student learning outcomes.

NYCTF and TFA teachers were required to keep reflective journals on their learning and teaching over the course of the methods course, which provided qualitative data of their attitudes toward learning and teaching mathematics. The learning journal had guiding questions such as: How has this course affected your teaching? What has been helpful? What are the most important concepts you've learned in this class? The teaching journal had guiding questions such as: How are your students learning? What challenges do you face? What successes have you had? Has your attitude toward teaching shifted over the course of the semester?

<div align="center">**Results**</div>

The first research question was answered using independent samples *t*-tests with significance at the 0.05 level. Findings revealed no statistically significant differences between NYCTF and TFA teachers on the mathematical content test, CST scores, attitudes toward mathematics instrument, and the PMTE and MTOE.

The second research question was answered through analysis of the learning and teaching journals. Analysis of the learning and teaching journals revealed similarities and differences between the two programs. For the learning journals it was found that both groups cited problem

solving and numeracy taught in the methods course frequently. Techniques for motivating student learning were discussed in both NYCTF and TFA journals. Both NYCTF and TFA teachers cited reflective teaching and literature critique reviews least often. While social justice was cited most frequently by TFA teachers, it was mentioned very infrequently by NYCTF teachers. While microteaching and learning about motivation techniques were two categories frequently mentioned by NYCTF teachers, TFA teachers rarely mentioned these. In both NYCTF and TFA methods classes teachers were expected to present a brief micro lesson that contained a motivator for the lesson to their classmates.

For the teaching journals both NYCTF and TFA teachers cited classroom management as the most frequently cited concern. However, it should be noted that while every TFA teacher reference to classroom management was citing a problem with classroom management, several NYCTF teachers mentioned classroom management as not being as problematic as they thought it would be. Also frequently referenced in both NYCTF and TFA journals were student motivation for learning, student attendance, and standardized state examination preparation as emphasized by their administrations. TFA teachers cited unsupportive administration frequently as a concern, whereas NYCTF teachers did not.

Discussion

It is commonly claimed by TFA that their candidates come from the most highly ranked and selective universities in the United States (TFA, 2010; Xu, Hannaway, & Taylor, 2008), and the implication is that those among America's brightest become TFA teachers. However, the findings in this study indicated that NYCTF and TFA teachers are statistically similar in terms of content knowledge and sense of efficacy. These results are quite surprising considering there is a common perception held by those working with the programs in New York that TFA teachers, while not staying in education quite as long at NYCTF teachers, have stronger mathematics content knowledge.

Since the results of this study indicate there are no differences found in several variables that measured teacher quality between the two programs, the implication is that it should not matter in which program teachers are selected based upon the results in this study. However, given results from prior studies that focused on teacher retention, perhaps NYCTF maintains an advantage over TFA using retention, a variable important in student success, as an important criterion for success. Generally, retention of NYCTF teachers has been comparable to the retention of traditionally

prepared teachers (Kane, Rockoff, & Staiger, 2006). NYCTF report that 92 percent of NYCTF teachers completed their first year of teaching, 75 percent completed at least three years of teaching, and half had taught for at least five years (NYCTF, 2011). Boyd, Grossman, Lankford, Loeb, Michelli, and Wyckoff (2006) reported that about 46 percent of NYCTF teachers stayed in teaching after four years as compared to 55 to 63 percent of traditionally prepared teachers. Sipe and D'Angelo (2006) found that about 70 percent of NYCTF teachers who were in their second year of the program intended to stay in education.

Upon completion of the commitment to the program, TFA said that nearly two-thirds of alumni stay in the field of education with about half of those alumni remaining in the classroom (TFA, 2011). This means that about one-third of TFA alumni stay in the classroom upon completion of their commitment. For those who stay in the classroom, about 90 percent of them teach in low-income communities (TFA, 2011). It is important to note that there have been critics of TFA's claims of teacher retention. Darling-Hammond, Holtzman, Gatlin, and Heilig (2005) claimed that upon becoming certified many TFA teachers leave the field, and Lassonde (2010) claimed that only 11 percent of TFA teachers reported planning to teach 10 years or more.

In the learning journals, teachers in both programs cited problem solving and numeracy frequently. More interestingly, social justice was cited most frequently by TFA teachers, but it was mentioned very infrequently by NYCTF teachers. This is an interesting finding given the emphasis on social justice in the school of education where this study took place. This finding may be due to the social justice perspective that motivates TFA teachers into their program. If social justice issues in the classroom are of high concern, then perhaps TFA teachers have an advantage over NYCTF, in this respect. This should be further investigated.

In the teaching journals, teachers in both programs cited, unsurprisingly, classroom management issues as the most frequently cited concern for new teachers, as documented in the literature (Cruickshank, Jenkins, & Metcalf, 2006). It was surprising, however, that NYCTF teachers found classroom management to not be as much of an issue, but TFA teachers found classroom management exclusively problematic.

While no differences were found for variables such as content knowledge, attitudes, and efficacy, it is important that we continue to understand differences in other important variables such as social justice orientation and teacher retention. Educational researchers must continue to investigate the quality of alternative pathway programs in preparing teachers for high need urban

schools. Since students in high need urban schools are often the ones who receive these teachers in the classroom, it is imperative that we ensure that these students are getting the highest level quality teachers they deserve.

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COMBINATORICS AND SAMPLE SPACE CONSTRUCTION: A LEARNING PROCESS

William McGalliard
University of North Carolina
at Greensboro
Wamcgall@uncg.edu

P. Holt Wilson
University of North Carolina
at Greensboro
Phwilson@uncg.edu

Using a mixed methods design, we investigate the relationship between prospective teachers' use of combinatorial reasoning to construct and make generalizations for the enumeration tasks necessary in defining sample space. Our findings suggest that there is an association between students' strategies for enumerating sample space and the ways in which they then generalize that enumeration. We present both quantitative and qualitative evidence for our conclusions and discuss implications for teacher education.

In recent years, there has been an emphasis on the importance of stochastic and probabilistic reasoning in curricular developments in response to national and international organizations (NCTM, 2000; Shaughnessy, 2007). One important probabilistic concept for students involves engaging in probabilistic decision-making. This understanding requires that students coordinate complex topics such as probability and systematic enumeration to construct sample space (Jones, Langrall, & Mooney, 2007). Coordination such as this forms the concept of the distribution, an understanding necessary for informed decision-making about uncertainty. However, research suggests that before the ability to make decisions with probability is possible, it must be preceded by other reasoning abilities such as combinatorics (Piaget & Inhelder, 1951). It is the purpose of this mixed methods study to elucidate the process that students use as they construct sample space and then seek to generalize their findings. It is our claim that understanding this process will better enable mathematics education researchers to understand one of the cognitive precursors to probabilistic understanding.

Research suggests that the probabilistic reasoning required for understanding distribution is necessarily preceded by other more deterministic concepts, specifically combinatorics (Piaget & Inhelder, 1951). Probabilistic reasoning involves thinking about a setting or context within a problem, which involves variation where an answer is not fixed or constant. However, deterministic reasoning involves thinking about a situation with a fixed endpoint. Both probabilistic and deterministic reasoning are necessary but are inherently different. Thus, the construction and understanding of sample space could be considered a precursor to probabilistic thinking since it involves systematic enumeration that involves deterministic reasoning abilities. An investigation of the process of sample space construction, which arguably forms a starting point for understanding

the probabilistic thinking that will follow it, is the motivation for conducting this study. In our work, we define sample space as the enumeration of all possible outcomes within a probability experiment.

Determining the sample space of a discrete probability event requires the use of deterministic reasoning to systematically and exhaustively generate all possible outcomes. The construction of sample space requires knowledge of how to systematically enumerate possible outcomes (Piaget & Inhelder, 1951), but research indicates that enumeration for even a simple random experiment is a nontrivial task for students (Jones, Langrall, Thornton, & Mogill, 1999). Some scholars point out that much of the cognitive complexity involved in constructing sample space stems from the dearth of experience students have with combinatorial reasoning (Batanero, Navarro-Pelayo, & Godino, 1997; Fischbein & Grossman, 1997). This suggests that systematic enumeration is a topic not well represented in the curriculum of school mathematics (English, 2005). Thus, we posed the following research question, which guided our study: What are the associations between the combinatorial enumeration strategies undergraduates utilize and the generalizations that they consequently produce when constructing sample space?

Theoretical Frameworks

Combinatorics is a branch of mathematics dedicated to the study of discrete structures or countable events. English (1991, 1993) offered a hierarchy for understanding the combinatorial strategies used by students (see Table 1).

Table 1.

Enumeration Framework (English 1991, 1993)

Strategy	Name	Description
1	Trial and Error	Students use a trial and error strategy.
2	Emerging Strategy	Students use some sort of pattern but it is not fully used or developed.
3	A Cyclic Pattern	Students use a cyclic pattern such as opposites that is fully utilized.
4	Odometer With Errors	Students hold one variable constant but fail to fully enumerate or over enumerate.
5	Odometer Complete	Students hold one variable constant and find a full enumeration

For us, these stages represent a way to operationalize the strategies that students use when they are engaged in a task that requires the construction of sample space. To understand the ways in which students generalize, we draw on the work of Lannin (2005). He suggests that the generalization process is not well understood and is underrepresented in the extant literature. His framework, depicted in Table 2, enables the categorization of generalization rules into a series of increasingly significant types.

Table 1.

Generalization Framework (Lannin, 2005)

Strategy	Description
Non-explicit	
Counting	Drawing a picture or constructing a model to represent the situation to count the desired attribute.
Recursive	Building on the previous term or terms in the sequence to determine the subsequent term.
Explicit	
Whole-Object	Using a portion as unit to construct a larger unit by multiplying (e.g. 3 sodas cost $8 so 9 sodas cost $24). There may or may not be an appropriate adjustment for over or undercounting.
Guess-and-Check	Guessing a rule without regard to why the rule might work. This usually involves experimenting with various operations and numbers provided within the context of the situation or problem.
Contextual	Construction of a rule based on information provided in the context of the problem and relating that to a counting technique.

This framework presents a model for the increasing sophistication of the different types of generalizations students produce in a variety of mathematical situations. In this study, we draw upon the work of English (1991, 1993) and Lannin (2005) to categorize the enumeration strategies and generalizations that students make as they construct sample space for a combinatorial task.

Methods

Design

We chose a parallel mixed methods design (Creswell & Clark, 2011), where there were at least two parallel and independent strands, which each has its own questions, data collection, and analytic techniques. First we collected quantitative data and statistically analyzed it to test for an association between enumeration strategy and generalization rule. Second, we qualitatively analyzed video data to provide a contextual explanation for the processes that students engaged in as they enumerated sample space and generalized about their enumerations for other similar situations. Finally, we mixed both sets of findings to create a more cohesive understanding of the problem.

Participants

There were 86 undergraduate students, enrolled in a mathematics course at a university in the southeastern United States, who participated in this study. The purpose of the course was to prepare them for entry into an elementary teacher education program. Then we purposefully selected a secondary sample of eight participants for follow up interviews. They were selected based upon whether they demonstrated growth, regression of understanding, consistently high, or consistently low understanding across the tasks. This criterion was important to assure that we reached data

saturation. They took part in a task-based interview (Goldin, 2000), where they were asked to recall the ways in which they solved the tasks from the prior phase of data collection.

Tasks

All 86 participants were given a series of three combinatorial tasks. Task 1 required the enumeration of all possible towers that were three blocks tall and made with two colors (Maher, Powell, & Uptegrove, 2010). Figure 1 is a visual representation, which is organized around the odometer strategy (English 1991, 1993), where one variable, the number of red blocks, was held constant, and allowed to occupy all possible positions.
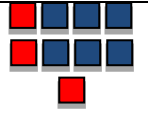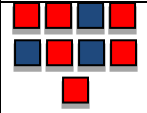
Three Block Towers				
Number of Red Blocks	0 Red	1 Red	2 Red	3 Red
Arrangement of Towers				
Number of Towers	1	2 3 4	5 6 7	8

Figure 1: Enumeration of Three Block Tall Towers

Task 2 required participants to build on these results and extend them to the construction of towers that were four blocks tall. Figure 2 is a representation demonstrating what Maher et al. (2010) referred to as the opposites strategy, where participants find an initial tower and construct its opposite making a pair of towers. They repeat this process until they have found all possible towers.
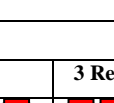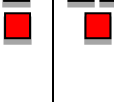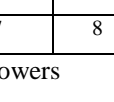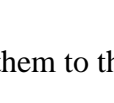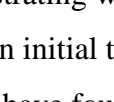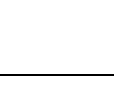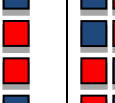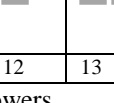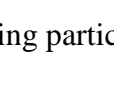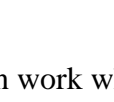
Four Block Towers								
Tower Pairs								
Number of Towers	1 2	3 4	5 6	7 8	9 10	11 12	13 14	15 16

Figure 2: Enumeration of Four Block Tall Towers

Task 3 called for the generalization of the prior two tasks, by asking participants to determine how many 100 block-tall tower combinations were possible.

Data Sources and Analysis

This study used two sources of data. The first consisted of written work where each of the 86 participants worked individually on the three tasks outlined above. The second source of data

consisted of video- taped interviews, where the eight purposefully selected participants were given back their individual work and were asked to discuss their thinking and ideas with one of the researchers.

Analysis occurred in three phases. The first phase was quantitative and involved categorizing the 86 participants' responses to Task 1 and 2 into one of the five hierarchical enumeration categories described in Table 1, categorizing their generalization rules according to Table 2, and used statistical analysis to test for an association between the enumeration strategy and generalization rule. We chose the nonparametric Fisher Exact Test, which SPSS calls the Linear-by-Linear Test of Association (Mehta & Patel, 2010) due to our small sample size. Had we chosen to use the chi-squared test of independence, the logical choice in a situation like this, we would have violated the sample size requirement of the chi-square test. The second phase involved using constant comparative analysis (Glaser, 1965) with open coding and discriminate case analysis to identify themes in the follow-up interviews. The third phase, was mixed and concerned using the qualitative themes, found in phase 2, to explain the quantitative findings found in phase 1.

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Findings and Discussion

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The purpose of this study was to determine if associations existed between the strategies participants used to enumerate a combinatorial task and the resulting generalizations they constructed. The quantitative findings are presented first, and show that an association existed. Then we present our qualitative findings, which are subsequently used to describe why an association was present.

Quantitative Findings

We began by generating contingency tables of the participants' enumeration strategies in Tasks 1 and 2 versus the generalization rule they produced in Task 3. We found a statistically significant association between the enumeration strategies used by participants on Tasks 1 and 3, ($p = .037$) and Task 2 and 3 ($p = .008$). However, both of these associations were weakly correlated but statistically significant, calculated using Kendall's tau-b (Mehta & Patel, 2010) and were both .231.

Qualitative Findings

We employed constant comparative analysis and identified two distinct, informative themes. The first was that participants seem to focus on finding explicit rules and formulas regardless of whether their rules make sense in the context of the problem. The second is that participants often formulate the odometer strategy in the context of the interview as they are discussing it or are

presented with evidence that the answer to one or more of the tasks was incorrect.

Theme One: Explicit Formulas. We first observed that participants showed the desire to find an explicit formula or rule. However, they did not connect this generalization to the context of the problem. For instance, Crystal formulated an initial generalization for Task 1 where she devised the formula $2 * 3 + 2 = 8$. When she applied this to Task 2, she found that her generalization of $2 * 4 + 2 = 10$ did not give her 16. She then pursued a different method, which used different numerical operations to help her find a generalization. To illustrate, Crystal described how she found the correct generalization.

> **Crystal**: So let's play with another method of getting bigger numbers. So that's how I came to the exponents and I was like 3^2 what's that? That's 9. Then I looked back and couldn't find another tower to make, so I knew that was wrong. So, I was like this is wrong, and I'll come back to it. Then I went here [Task 2] and was like this is 16. So, why don't I flip the exponent? But it doesn't use the grouping that I need to use, because if I were to do 3^2 here [Task 1] I don't get 8. So, that can't be the right rule. So, then I was like what if I took 2^4? So, then I was like 2^3, and I got the 8. That matches with this [Task 1] and 2^4 is 16 and matches up with this [Task 2]. So, that was my number. So, that's how I got the rule. Then the exponent is how many blocks high the tower is. So, I was like this is 3 blocks high and this is $2 * 2 * 2$, which is 8.

Theme Two: Formulation of the Odometer Strategy. The odometer strategy, as described by (English, 1991, 1993), is an enumeration strategy that consists of holding one variable constant and letting another variable vary to find a complete and systematic listing of all possible outcomes. Not all participants immediately used the odometer strategy in a productive way. During the interview, some formulated the strategy by discovering through discussion and experimentation that their enumeration did not yield their anticipated answer. Once we provided them with the opportunity to explore their generalization, they invented the odometer strategy as an efficient and sophisticated way to enumerate a combinatorics problem. For example, during the interview process, Cathy developed the odometer strategy as she described her work. However, she described the odometer strategy in terms of ratios and rotations.

> **Researcher**: So you are separating some of these towers off.
> **Cathy**: Yeah, by ratio. [RRRR and BBBB] Those would be all of one color.
> **Researcher**: So, these are separated off because…? [BRRR and RRRB]
> **Cathy**: They're the 3 to 1. This is where I don't think that's the total number [She's referring to her previous answer of 14 total towers]. Ok more about that rotation thing. [The blocks are arranged: BRRR RBRR RRBR RRRB] For the three to one I'm moving it from here to here to here to here [She indicates the blue block moving down].

 She realized that she did not have all the combinations of 4-block tall towers and therefore

developed the odometer strategy as a way of convincing herself that she had fully exhausted all possibilities. Cathy indicated at the end of the interview that she had struggled with the enumeration of 4 block tall towers made with 2 colors because she continually had to check back and verify that she had all possibilities prior to her construction of the strategy. Once she developed the strategy, however, she was confident that she had found all possible four block tall towers.

Mixed Findings

The strength of a mixed methods study is that both qualitative and quantitative analyses can used to help answer research questions. In this study we used the qualitative themes outlined above to help explain the statistically significant associations found in the quantitative analysis. The qualitative findings suggest that as students engage in a combinatorial task they see the need for organization and often construct the odometer strategy, and try to identify a formula that will enable them to generalize more easily. Cathy, as noted earlier said she struggled with Task 2 and needed to organize her enumeration in a better way. Thus, she developed the odometer strategy. Therefore, an association exists between the enumeration strategies that participants used and their subsequent generalizations because they saw the need for organization as they progressed.

Crystal demonstrated with her explanation of how she found her formula why there was a low, yet significant, correlation between enumeration strategy and generalization rule. She clearly wanted to find an explicit generalization but did not approach it from a contextual standpoint. Instead, she found a formula that would work for the observations that she made and then decided what the contextual explanation would be after the fact. She demonstrated that while participants could successfully generalize they were not necessarily mathematically making sense of their generalizations as they constructed them. This qualitative finding provides an explanation for the observed weak correlation.

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Conclusions

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Our study provides information about one cognitive precursor to probabilistic reasoning, the construction of sample space. Our findings suggest that certain enumeration strategies, such as the odometer strategy, assist prospective teachers when generalizing their understandings of the construction of a discrete sample space. When engaging prospective teachers in similar tasks, teacher educators should encourage the development of such strategies to support the development of the ability to generalize. In particular, our research suggests that candidates may not consider the context of the problem when generalizing. Instead, they tend to engage in what Mason (1996)

referred to as local tactics. In this situation students seek a generalization or formula based not on the context of the problem but instead generalize based on what works in the specific situation that they are investigating. This suggests that teacher educators should provide prospective teachers with opportunities to use certain enumeration strategies and encourage them to think about the problems in terms of its context. Our study provides insight into the construction of discrete sample spaces, which is a cognitive precursor to probabilistic reasoning.

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USING DESIGN EXPERIMENT TO EXPLORE MULTIPLICATION IN A MIDDLE GRADES MATHEMATICS METHODS COURSE

Sarah Smitherman Pratt
University of North Texas
sarah.pratt@unt.edu

A design experiment was conducted in a middle grades mathematics methods course to examine pre-service and lateral entry teachers' understandings of multiplication. Algeblocks *were used during data collection, and word problems were written each session as a way to analyze and self-analyze conceptual understandings of multiplication of integers and of binomials, with the result of improving the participants' mathematical knowledge for teaching (MKT). The data collected demonstrated that the design experiment was a successful tool in examining their process for learning and that the participants shifted in their engagement in mathematical tasks that improved their ability to think more flexibly.*

The middle grades curriculum includes a focus on the understanding of the set of real numbers and operations using real numbers, and the middle grades curriculum also includes a focus on understanding algebraic expressions. These two main ideas merge in the curriculum well when the concepts of negative integers and polynomials as well as operations using integers and polynomials are the topics for instruction. These concepts are difficult for many middle grades students to understand. Teachers often supply mnemonic devices—such as FOIL to multiply two binomials—to assist students in memorizing the procedures by which the operations follow. These devices can hinder growth in understandings of what the operations actually mean with respect to understanding numerical and algebraic structures, and this may lead to problems for students in their future mathematical careers.

One particular operation, multiplication of two negative integers, is difficult to conceptualize; even more so, it is difficult to engage others in exploring a meaningful investigation to discover why a negative integer times a negative integer results in a positive integer. Algebra tiles can be used to demonstrate multiplying a negative integer times a negative integer using an area model. While I agree algebra tiles offer an image with which understandings can be elucidated, I believe there are limits to their usage. I prefer *Algeblocks* because they display a different representation than algebra tiles when examining multiplication. The blocks are placed on a quadrant mat, which represents the four quadrants of the Cartesian plane. While algebra tiles remain in one quadrant and tiles are flipped over when multiplying a negative times another value, *Algeblocks* use four quadrants, thereby connecting to an important concept in mathematics, namely the Cartesian plane. Thus, multiplying two integers using *Algeblocks* can assist understandings multiplication as

grouping to multiplication as area, and using the Cartesian plane as the intersection of two number lines shifts from a one-dimensional to a two-dimensional model. The use of the Cartesian plane became a significant point of reference with the participants in this study and was instrumental in improving understandings of multiplication of integers.

Theoretical Framework and Related Literature

This research study is informed by two main perspectives in educational research. The first is *Mathematical Knowledge for Teaching* (MKT), as developed by Deborah Ball and Hyman Bass (2003). The *MKT* model relies on Shulman's (1986) pedagogical content knowledge, but Ball & Bass (2003) incorporate subject matter knowledge as another key component in their examination of what it means to be a mathematics educator. I utilize this framework in my research and in my pedagogy. As a mathematics teacher educator, I focus on the development of *MKT* as a way to encourage them to listen (Davis, 1996) to their students. Before they can listen well, though, they have to have flexible ways of thinking about mathematics. In order to develop flexible ways of thinking about mathematics, an implementation of tasks that enrich understandings rather than reinforce procedural knowledge must be undertaken. These tasks develop eruditions with respect to particular concepts, and they should encourage investigation, questioning, struggling, and arguing offer opportunities for teachers to develop their own mathematical knowledge for teaching (Ball & Bass, 2003; Ball, Thames & Phelps, 2008). *MKT* requires a more complex and dynamic understanding of mathematical concepts than merely procedural knowledge, so the tasks I choose must be rich with potential.

My second theoretical perspective, which I believes relates nicely with *MKT* as well as my methodology of a design experiment, is my understanding of teaching and learning from a complexity sciences perspective (Pratt, 2008; 2011). Education researchers who draw on complexity science believe teaching and learning can be emergent, dynamic, and alive (e.g., Doll, 1993; Davis, 1996; Fleener, 2002). Taking this perspective allows me the freedom to plan but know that I cannot predetermine. Thus, I engage in the task of developing both my own mathematical knowledge for teaching and encouraging others around me to do the same. In the current mathematics education literature, there is little data on *MKT* for middle and secondary teachers. This research study examines the MKT of students enrolled in a required course to achieve middle grades mathematics certification with respect to multiplication of integers and polynomials.

Methodology

To investigate how pre-service and lateral[1] entry middle grades mathematics teachers understand what it means to multiply two binomials, I engaged in a design experiment (Cobb et al., 2003; Gravemeijer & Cobb, 2006). I chose design experiment because "the purpose of design experiments is to develop theories about both the process of learning and the means designed to support that learning" (p. 18), and from a complexity sciences perspective, this method of design experiment allows for emergent adaptation while in the midst of the study. In the spring of 2008, the first design experiment was conducted, with 13 participants who were enrolled in the only middle grades mathematics methods course required by the university and state licensing agency. Data was collected during one hour of instructional time on three occasions over a two month period using video, audio and written responses. Participants were of mixed gender, race, age, and educational background. The participants were given a series of tasks each week on three distinct occasions and asked to work through them in pairs as the entire group moved forward collectively. In addition to myself as instructor of record, a colleague assisted in observing and collecting the data during the experiments. We discussed what transpired after each session to consider how to adjust for the following session, which is in line with design experiment methodology (Nickerson & Whitacre, 2010).

The focus over the series of tasks was to conceptualize what it means to multiply, first integers then monomials then binomials. In particular there was an emphasis on understanding multiplication as grouping, arrays, and area (Wheatley, 1992). The first task, before the study was conducted, was for the participants to write a word problem that requires the multiplication of two binomials. This type of task is adapted from Liping Ma's (1999) investigation of teachers' understandings of division of fractions (and is something we did previously in the course with respect to dividing fractions). By requesting an invented word problem, I wanted to draw out the participants' understandings, misunderstandings, or misconceptions of multiplication of binomials. The request to write a word problem was repeated throughout the experiment to allow for participants and myself to examine and reflect on changes in personal understandings, an iterating cycle throughout the sessions.

Each week the series of tasks required the participants to utilize *Algeblocks* to aid in their

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[1] In this study, a lateral entry teacher is a participant in an alternative certification program who obtained a degree in something other than education and is now working toward certification through a state-approved route.

understanding of multiplication. During the first day, we moved from the multiplication of two integers that are positive to one positive and one negative, to two negative integers. For each pair of numbers, the participants were asked to provide the following information: 1) Record your *Algeblocks*; 2) Explain your process; and, 3) Write a mathematical solution. These three pieces of information were designed to assist the participants in making the transition from the hands-on task of manipulatives to the formalized mathematical knowledge they need to acquire. The second day, we continued with the multiplication of integers then moved to multiplying integers times monomials. The information requested during the second day was amended. (In line with the method of design experiment, I added to the information requested for each problem because I realized that if I wanted them to write a word problem at the end, I needed them to tell me more in the process.) With each new task, before engaging in the *Algeblocks*, the participants were asked to write, in words, what they believe the problem is asking. Following the three pieces of information as listed above, they were asked to write a "real world" context for the expression. (*Figure 1* provides a sample of the series of problems the participants were asked to complete.) This last piece of information provided the most useful data and elucidated many misconceptions and struggles among participants. On the third day, they provided the same pieces of information as day two, transitioning to the multiplication of monomials and binomials.

Problem #1:   2 - - 4

Explain, in words, what you think this problem is asking:

Record Your *Algeblocks*:

Explain Your Process:

_____
_____
_____
_____

Provide a Mathematical Solution:

_____
_____
_____

Provide a "real world" context for this expression:

Figure 1: Day 2, Problem 1.

Statement and Discussion of Results

Over the series of tasks related to the multiplication and the use of *Algeblocks*, two important aspects of the study emerged. The first is that the data collected demonstrated that the design experiment was a successful tool in examining the participants' processes for learning. The participants engaged diligently in the tasks throughout the study. They also challenged preconceived ideas as we progressed. The first question that was raised, which I believe is an important one, is why the third quadrant on the *Algeblocks* quadrant mat is labeled "positive." This was what I wanted them to understand, that it is positive, but as they brought up this question, I realized that often in mathematics we give students assumptions without opportunities for them to question and struggle why a particular definition or axiom must be included. We did not complete many tasks during the first day of data collection because we spent so much time trying to address this question. My attempts to explain were not accepted by the group. This was a good place to be, I thought, because I realized they really wanted to know and understand what it means instead of continuing to play the game of accepting "truths" by the teacher. I highlight their struggles and conclusions with respect to the multiplication of two negative integers because this was a conceptual turning point for the group.

Because of their challenge to me, I adapted the questions I asked, specifically to find out what they think the problem is asking before even handling the *Algeblocks*. This was my way of trying to listen better to what they already understand so together we could recognize if they did not understand. The "real world" context provided the most stimulating conversations during the class time. I had one participant provide an analogy of SCUBA diving for multiplying a positive times a negative, which connected to another. However, when we moved to a negative times a negative, the one-dimensional model did not connect to this analogy for anyone. Simultaneously, another participant realized that the Cartesian plane is the intersection of two number lines, and that the area model of multiplication using *Algeblocks* demonstrates the two-dimensional relationship of the plane. Finally, another participant came up with the example that if "you" "owe me" money, that is money I have coming to me, which is positive. (The *x*-axis is "me + and you –," and the *y*-axis is "I get + and I owe –" in this analogy.) The group seemed to collectively agree, after much discussion and arguing, that this example was acceptable as a word problem with a negative times a negative, and could explain why the third quadrant is positive.

This leads to my second significant finding, that the participants developed their abilities be more flexible in their mathematical thinking. In their arguments around the acceptability of word problems, they were able to analyze various problems and determine if they were, in fact, mathematically sound. The skill to be more flexible in their thinking is a focus in *MKT*, for it allows teachers to be better listeners of students' conceptions of mathematics.

I knew that the final question about analyzing their original word problem would pose problems for them unless we continuously attempted to create connections to the real world with each expression. The word problems were excellent sources for whole class conversations about the concepts. Even more telling was when examples could not be provided. I was encouraged when I read their final word problems, for I found that they did improve their conceptual understandings of multiplying integers as well as demonstrate a significant contrast to their original ideas about what it means to multiply two binomials. This connects directly to thinking about their own opportunities for them to engage their own students differently to investigate mathematical ideas, knowing that the students need time to process and articulate what they are thinking (which are the NCTM (2000) *Process Standards*).

The process of learning during the design experiment was perhaps the most significant for the collective group. On the final day the focus of the experiment was on understanding how multiplying a binomial with another binomial is really just the distributive property (as opposed to only knowing how to FOIL). As we progressed through the tasks of multiplying an integer with a binomial, a monomial times a binomial, and a binomial times a binomial, the participants became animated in their examples of what these expressions are asking and argued among themselves about their ideas. As each expression was offered, the task of writing real world contexts for binomials caused perturbations for many of the participants.

At the conclusion of the experiment, the participants received their initial word problems. They were given time to read, reflect on, and chose to keep or rewrite their word problems. All but one rewrote their problem. The task of reflecting on their original work was powerful, for many recognized the growth in their understandings of integers and operations with integers and polynomials. Furthermore, their word problems were much more realistic and easy to follow, which demonstrates more proficiency with mathematical concepts in connection with the "real world." For example, one participant's original word problem was "Fred had (I have no idea at all)," but the rewritten problem was, "You have a square garden. You add 2 inches to the width

and 3 inches to the length. What is the new area?" and included drawings (*Figure 2*).

Original

x

x

x + 2

x + 3

(x + 2)(x + 3)
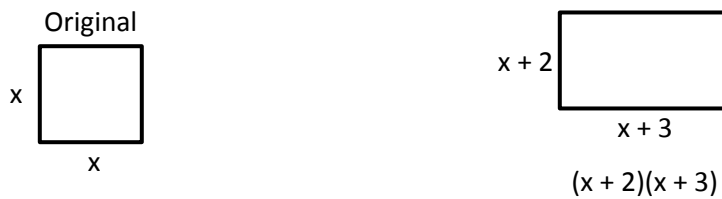
Figure 2: One student's drawing to accompany the constructed word problem.

The transformation of conceptualizing multiplication as an area model for this participant is remarkable, for it demonstrates a move from an inability to articulate a binomial to a real world problem that integrates binomials.

Both the process and the improvement of *MKT* can be highlighted in one participant's final project for the course. She was a middle grades special education teacher who pursued how blocks and the area model could be used for division, a concept that her students struggle to understand because of the complicated nature of the division algorithm. After some discussion in our classroom and investigations on her own, she took the idea back to her classroom. (A more detailed explanation of her investigation can be found in Richardson, Pratt & Kurtts, 2011.) Overall, she recognized how complicated the division algorithm seems, and she engaged her students in exploring the actual concept of division through grouping then followed with an area model. She shared her investigation with the class, and it was an inspiration to us all about how multiplication and division are inversely related and strongly connected. Her presentation was a great example of how she developed in her own understanding and teaching of mathematics (*MKT*).

Conclusion

By engaging in a series of tasks designed to explore multiplication, I challenged the notion that multiplication is merely repeated addition or that using algorithms will lead to conceptual understandings. Understanding multiplication as grouping, arrays, and area (Wheatley, 1992) leads to potentially richer conceptual understandings of more complicated mathematical topics in number theory, abstract algebra, and geometry. One aspect I learned from this experiment is that before participants can engage in understanding multiplication of integers, first the understanding of multiplication should be addressed. Therefore, when I taught a new course in the following semester, I implemented an investigation using base-10 blocks to multiply whole numbers and *Algeblocks* for monomials and binomials, remaining only in the first quadrant before shifting to all

four quadrants.

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JOY'S DESCRIPTION IN A PROBLEM CENTERED LEARNING SETTING

Sandra Davis Trowell
Valdosta State University
strowell@valdosta.edu

One student, Joy, describes her experiences in a problem centered learning mathematics classroom. The student is in an undergraduate senior mathematics problem solving course which she finds to be different from her traditional mathematics classroom. Joy very acutely describes this classroom and how she came to think of problem solving. Her analysis is important in making sense of factors that influence mathematics learning.

This paper describes a student's beliefs, actions, and interpretations of a problem solving course - Problem Solving in Mathematics - in which she was a student. The student, Joy, was interviewed throughout the semester long problem solving course as part of an investigation of the social norms that were negotiated in the course (Trowell, 1994). Joy was a senior mathematics education major who excelled in all of her classes. She was an extremely reflective person.

Research Procedures

The explanations that were constructed to explain Joy's beliefs, actions, and interpretations were from an interactionist perspective (Blumer, 1969; Bauersfeld, 1988, 1991). A mathematics classroom was not viewed as a "medium" through which participants passed to form their behavior. Instead the participants acted according to their meanings as they interacted with others in their classroom. In attempting to make sense of the negotiation of social norms in Problem Solving in Mathematics, it was important to listen to many different participants and provide opportunities for each to elaborate upon their experiences in this class. By video recording each class session, there were opportunities for looking back at classroom actions and interactions. In addition, field notes for each session were taken. Immediately following each class session, the instructor shared his ideas, reflections, and thoughts concerning the class with these interviews being video recorded.

Approximately four weeks into the semester four students were chosen to be interviewed regarding their experiences in PSM. These students were seen as being different from each other in their mathematical sophistication and their classroom interactions. Only the student and a mathematics educator were present for these interviews. The students had opportunities to elaborate upon their classroom experiences, problem solving, and beliefs about mathematics and mathematics classrooms. By considering the instructor's, students', and a researcher/observer's views different experiences were available from which to begin to figure out the negotiation of classroom social

norms. To facilitate in depth analysis, some excerpts were transcribed and examined in more detail. The intention was to look for recurrent patterns (Voigt, 1992). The analysis involved making inferences based upon the actions and interactions of the various participants in PSM.

Joy

This paper describes Joy – one of the four students chosen to be interviewed - and her experiences in this classroom. Joy was one of the strongest mathematics students in the class Problem Solving in Mathematics. She quickly figured out the possibilities as the social norms were being negotiated. Joy was the first one to ask the instructor, Dr. M, to elaborate on the assessment procedure for this class. She participated in whole class discussions in a quiet and consistent way. Joy frequently chose to focus on particularly challenging problems while still feeling the necessity of completing all problems.

During interviews, Joy was very open and quite willing to share her thoughts concerning Problems Solving in Mathematics. She seemed to become a part of the research with me as she attempted to reflect deeply upon her classroom experiences. I even had to cut one interview short to remind Joy of a class that she was scheduled to attend; she had become so involved in telling me about class that she had forgotten the time. I rarely had to question Joy after the initial interview. I simply asked her to tell me about class and she continued from there. What follows will be my description of some of Joy's insights and beliefs about problem solving as a result of her experiences in this course. Joy was descriptive in her interviews and for that reason there will be several lengthy quotes from these interviews.

Problem Solving in Mathematics Course

This Problem Solving in Mathematics course was approached with a problem centered learning focus that was student-centered in nature, since a traditional teacher-centered classroom was seen as providing limited opportunities for students to become problem solvers. Students in a problem-centered learning environment are seen to have more opportunities to become more mathematically and mentally fit. Consequently, instead of simply working on exercises, students were asked to solve a variety of problems in ways that made sense to them. Students were provided with opportunities to explore and solve problems.

> Problem centered learning has three components; Tasks, groups, and sharing. In preparing for class a teacher selects tasks which have a high probability of being problematical for students - tasks which may cause students to find a problem. Secondly, the

students work on these tasks in small groups. During this time the teacher attempts to convey collaborative work as a goal. Finally, the class is convened as a whole for a time of sharing. Groups present their solutions to the class, not to the teacher, for discussion. The role of the teacher in these discussions is that of facilitator and every effort is made to be nonjudgmental and encouraging. (Wheatley 1989, pp. 15-16)

In this problem-centered classroom the teacher was engaged in doing mathematics/problem solving with his students. He did not view his responsibility as persuading the students to "see it his way."

Problem Solving for Joy

Joy said that she enjoyed problem solving. She related an account of her first encounter with problem solving during her freshman year of high school. She talked of her Algebra I teacher emphasizing problem solving and the fun that she had had with "word problems" since this experience. She was not intimidated by such problems and said that she enjoyed being challenged.

During every interview, Joy would recount how she was working on a problem or how she was thinking about a problem. She was very focused on solving the problems and talked of her explorations. She described the key to problem solving as "persistence." She said that in telling someone about problem solving she would tell them to just be patient and persist. Joy pushed for her solutions to be reasonable and to make sense to her.

> **Joy:** I feel like if you're solving a problem and you realize you made a mistake and you go back and you reanalyze your problem then you had to think to figure out what was wrong - I mean, you had to actually realize and think that this is unrealistic, you know. Some people don't recognize it as thinking though.

Problem solving was a personal sense making activity for Joy. It was not simply applying a procedure and having the answer pop up. She said that "thinking", exploring, and analyzing the reasonableness of your solution was an important part of problem solving.

Joy said that at first her goal was to "get an answer," but by the end of the semester, she said that developing a successful problem solving plan became her goal. Her focus shifted from "getting an answer" to figuring out how to solve the problem. Joy said that when "you realize that you can solve it, it's not a problem anymore." Joy continued to push herself to find solutions for all of the tasks and became fascinated with the heuristics of problem solving.

Joy also came to appreciate other student's solutions. She said that she worked problems in ways that were "meaningful" to her, but that learning about other solution methods for a particular problem was valuable. As an example, Joy described Robert's way of solving a particular problem as "elegant" and "simple". She had spent a long time working through a system of equations, but when Robert explained his way of thinking about the problem, she had been very impressed. Joy said that she would begin solving a problem using strategies that had previously worked for her, but when she got stuck, she would remember some other strategy that had been demonstrated in class and would try it. Joy also pointed out that listening to other people share their ideas makes one realize that people think differently about their problems.

> **Joy:** ... I would think twice before doing a problem like that again. Which is kinda neat, but then if I got stuck I would go back to my old ways, but at least I tried something different. It helps - I think it helps me as I'm going to be teaching other students to problem solve to realize - I mean it makes you really realize that they are really going to see it differently and when they're asking you for help you don't need to say, "No look. Here's this equation, this equation - now solve it!" You say, "Oh you see it this way! Okay, now let me think about it for a minute." And just pray that you see it their way and work with them their way and not through your way. 'Cause they're going to have a whole 'nother way.

For Joy, sharing ideas again pointed out that mathematics was a personal activity. She felt that hearing these ideas was also beneficial to her as a future mathematics teacher not wanting to impose her ideas upon others.

Problem solving for Joy was not memorization of the way someone else had worked it, but something that she experienced. She said that problem solving becomes easier "just from experience." Joy did not see problem solving as something that someone could give to you.

> **Joy:** A lot of times I don't want to know how they worked it before. I don't mind them giving me a hint - like saying similar triangles, but I try to tune out everything myself if I haven't worked it yet myself because I want to have the opportunity to, you know, I want to see if I can get it. Because I found that I can't memorize things. I don't remember how I worked it if someone just tells me. I never, ever remember. I have to work it myself before I remember, so. I think it's fun. I enjoy the class.

Joy did not want someone to tell her how to work a problem. In order to learn mathematics, she wanted to find the solutions through her experiences.   She wanted to do it herself as this was what she found meaningful to her.

Joy described problem solving as "important" - she differentiated problem solving from conventional mathematics classroom activities.

> **Joy:** I think problem solving is important. It makes you think - it's not (in a sing song voice) 2+2 is 4, 4+4 is 8, here's how you add the natural numbers.   Add an x here, subtract an x from both sides, life's fun, here's your problems, have a nice day. There's 30 problems, do 1- 50 odd, turn them in tomorrow. You can teach them algebra through problem solving. I think problem solving is real important. It applies - like I learned it in algebra I. It made sense in the algebra I. ... If they learn all the algebra but they don't learn how to apply it like word problems, they're not going to use it in real life. Like I use it - when dad was trying to build this....I'm like, "oh dad look, it's just like this." .... It makes you think. ... I think it stimulates you and it combines all of math together. In one problem they can do algebra, geometry and trig and they can see a purpose in all of that. So it doesn't separate it into - here's (moves her hand to show different categories) and algebra has nothing to do with geometry....   They need to see that there's a connection between these.

For Joy, the various domains of mathematics became connected and she emphasized that problem solving was something that she did all of the time - in and out of school.

When trying to solve a problem, Joy stated that she would refer to other textbooks. It was comfortable doing so. Looking in other resources was natural to her as she attempted to find solutions to her problems.

Small Group Experiences

While Joy believed that she profited in hearing other people's solution, she also felt it was important to initially work individually as her small group came to do. During the course there were frequent times when the students were asked to work in their assigned small groups on various problems. She described her small group time as when she had an opportunity to see how the others think about their problems after working on their own. This strategy also seemed to work well with the others in her group - Robert, Gail, and Marcus.

Joy: In the groups everybody talks 'cause you kinda have to or the group doesn't do anything. But, um, I think in the groups all we're doing is just comparing the way we did it. I know, I've worked with, like each person - like they might have a solution half-way and when they present it to the rest of the group, the rest of the group sees it and so that they can solve it. And that's fun, it's just - I feel like all we're doing is just solving problems, I mean, but it's fun.

Joy saw her group time as a chance to solve problems and present ideas. She figured that the other groups were doing the same thing as her group. Joy also noted that on the occasions that a member of her small group was absent, the group was different.

Joy believed that Dr. M assigned the groups homogeneously. She was reluctant to say this and chose her words carefully as she did not want appear to be putting others down.

Joy: I think he put people together who were equal - like the ability to solve problems was close to being the same. You know, not necessarily they could solve the same kind of problems, but like you have a variety but you still have a balance. You don't have one person who can't solve problems who's going (Joy opens her mouth and sits there looking dumbfounded) while everybody else is going (Joy acts as if writing on paper.) - so that in that group those three people - don't spend the entire time just explaining to one person. That if we get a problem in our group we - like in our group we can all see a certain way to solve it ourselves and we each usually solve it and then we discuss our answers. Like in maybe some of the other groups they - maybe people don't always get the answers - he paired them so that they would complement each other so they could solve the problems. That's my thought. I kinda feel like that's what happened. That's what I would do so that one person doesn't feel unequal to someone else.

Joy felt it was important for people not to feel "unequal" in groups. She said that Dr. M "paired us with people he thought we would do well with." Small groups were not little classrooms for more sophisticated persons to tell or explain to others.

Joy believed that during small group time, students were involved in discussing ideas about their problems. Her group chose to spend some time thinking on their own about the problems and then returning to share with each other about their thinking. Small groups meant collaboration with her peers.

Summary

Joy was a strong problem solver who valued all ideas. She said that problem solving required time and "persistence." Joy saw problem solving as a personal activity which could involve ideas from any branch of mathematics and she hoped that as a future mathematics teacher her students would become as excited about problem solving as she was. Joy believed that mathematics was not something that a "knowledgeable person" could give her, but that through perseverance and experience she could make it her own. This course also helped her recognize the necessity of listening to others ideas and thinking as problem solving was personal and was supposed to make sense to each individual. She sought to have the attitude of listening to students rather than imposing her ideas upon them.

It became evident through the interviews and her classroom interactions that Joy made a concerted effort to make sense of mathematics and mathematics classes. She believed that mathematics was a subject she could understand if she worked sufficiently hard. While she was an independent person in all respects, she believed that collaborating with others and listening to other ideas was important for her in problem solving. She saw collaboration as important in testing her reasoning rather than getting help in solving problems. Joy became autonomous in her problem solving.

Joy struggled with her own desire for everything to be "perfect," yet she thrived and flourished in Problem Solving in Mathematics. In listening to Joy we recognize that mathematics problem solving is a personal activity and mathematics classrooms should be places for students to explore, listen, and defend their mathematical ideas and relationships.

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DEVELOPING THE PRACTICE OF TEACHER QUESTIONING THROUGH A K-2 ELEMENTARY MATHEMATICS FIELD EXPERIENCE

Catherine Stein Schwartz
East Carolina University
schwartzca@ecu.edu

This paper presents findings from research on a field experience designed to help elementary preservice teachers learn how to use questioning during formal and informal interviews to analyze student mathematical thinking in K-2 classrooms. Preservice teachers were specific and accurate in identifying a mathematical goal and analyzing student thinking when responding to a video-taped interview, but were less successful in their explicit discussion about the practice of teacher questioning. More research is needed to understand tasks for preservice teachers in early field experiences that will help them develop intentional use of teacher questioning to facilitate student thinking about mathematics.

Field experiences are a mainstay in traditional preservice teacher education. Preservice teachers (PTs) report the time spent in classrooms during internships to be the most influential and useful part of their preparation programs. However, sometimes field experiences send messages that are opposite what is advocated for in mathematics methods courses and by research (Darling-Hammond, 2005). Recently mediated field experiences (Zeichner, 2010) have been powerful in helping PTs make sense of what they are doing and seeing in the mathematics classroom. Concurrently, researchers have focused on the notion of high-leverage practices (Ball, Sleep, Boerst, & Bass, 2009). Knowing that the complexity of teaching takes time to develop, high-leverage practices are those practices that are most essential to the work of teaching in terms of impact on students and that are most likely to be easily accessed by beginning teachers.

Planned revisions to an existing K-2 mathematics practicum created an opportunity to deliberately design a mathematics field experience around a single high-leverage practice, teacher questioning in formal and informal interview settings. This practice seemed particularly appropriate for several reasons. First, effective teacher questioning is widely acknowledged as an essential part of mathematics teaching (Carpenter, Fennema, Franke, Levi & Empson, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004). And yet, preservice teachers are not intuitively competent at questioning (Ralph,1999a, 1999b). Moyer and Milewicz (2002) found that PTs' experience interviewing students allowed them to recognize and reflect on effective questioning, but they still were often *checklisting* or falling back to instruction rather than probing for student thinking. Second, interviewing is a primary form of assessment in grades K-2 and therefore an ideal focus for a K-2 field experience. Finally, the preservice teachers in the practicum are typically juniors taking one of

their first education courses. Working on teacher questioning with individuals and small groups of students is more manageable for early beginners in the field than other potential and important high-leverage practices such as orchestrating whole class discussions (Ball, et al., 2009).

Experiences were designed for preservice teachers to view the practice of questioning through the lens of the instructional triangle. The three elements that interact with each other in a context to promote mathematics learning are the mathematics, the teacher, and the students (NRC, 2001). Throughout the semester, PTs were asked to focus on a mathematical goal, listen to student thinking about the goal, and decide on a teacher action in response to student thinking.

This study reports on data collected during the first pilot of the revised practicum. The purpose was to see if the preservice teachers in the pilot group differed from those in the existing practicum in terms of mathematics content knowledge and the practice of teacher questioning.

Methodology

The participants in this study were 50 prospective teachers enrolled in two sections of a required grades K-2 mathematics and methods course taught in the beginning of an elementary teacher education program at a large state university. The one-hour practicum was completed concurrently with the course. Participants were divided into the different practicum groups simply based on the section for which they registered. They had no knowledge of the different practica experiences ahead of time. The two sections of the course were taught back-to-back by the same instructor on the same days of the week. Preservice teachers in the two sections differed only in their completion of either the existing or revised practicum. They were exposed to the same material during the university coursework.

Existing Practicum

In the existing practicum, PTs were placed in individual classrooms at multiple schools in multiple school districts. They were sent to the placement with a list of assignments to be completed during 15 hours of time spent in their assigned classrooms. Individual student interviews were included in the list of assignments. PTs were to interview an elementary student eight times across the semester. Suggested activities were provided for the first four interactions based on grade level, and usually consisted of a single open-ended word problem for each session. PTs then decided on the content and tasks for the second four interactions.

Tasks, reflections, and student work were turned in at the end of the semester with the other required assignments. One reason for revising the practicum is that the student interviews, intended

to help PTs analyze student thinking often turned into tutoring sessions at the request of the classroom teacher. The focus often shifted from student thinking to teacher telling.

Revised practicum

Preservice teachers in the revised field experience were placed in one of three classrooms where they conducted structured interviews with K-2 students using a web-based interview assessment focused on early number sense (Richardson, 2003). Then they worked collaboratively to analyze data and determine student groups. They each taught a small group of two or three students in the subsequent four sessions, meeting during university class time between each session to debrief and jointly plan the next lessons based on student work. Preservice teachers were asked to state their mathematical goal for the lesson, explain what specifically they were looking for as evidence of student thinking, and list the questions they planned to ask with anticipated student responses. They submitted a written reflection after each lesson detailing their students' thinking and their own questioning. Finally, PTs repeated the structured interviews with students at the end of the semester and wrote a final paper about the entire field experience.

Measures

Data collected from PTs in the current and revised practica were analyzed for differences in mathematics content knowledge for teaching and in the practice of teacher questioning. Both measures were given at the end of the semester.

Content knowledge was measured with questions selected from the Mathematical Knowledge for Teaching measures (Hill, Schilling, & Ball, 2004). Twelve items from were selected from the Elementary Number and Operations forms of the test. Twenty-two questions were initially selected to match the content of the course, namely those topics in number that are typically taught in a grades K-2 classroom. These topics included whole number addition and subtraction, place value, equality, and patterns and functions. Then because the MKT measures are designed for use with practicing teachers and the use in this study was with early preservice teachers, the 22 questions were narrowed to include only those with a difficulty level less than 0.555. A two-sample t-test was conducted to compare the mean scores of each group. No difference between the mean scores was expected since both groups completed the same methods coursework.

Teacher questioning was assessed using a video interview assessment. Preservice teachers watched a seven minute interview of a second grade student, Amy, from a training video used in professional development with practicing teachers when the North Carolina K-2 Assessments were

first introduced. The interview focused on the concept of tens and ones as noted by the title of the segment on the training video. This title slide was edited out of the video so that the PTs would not have any preconceived notions about the interview. PTs watched the interview online and then responded in writing to three open-ended prompts that were created based on the instructional triangle (NRC, 2001):

1. What mathematical content is the teacher trying to assess? Explain how you know. (mathematics)
2. What does the student understand? What does the student not yet understand? Be specific. Use what Amy says and does in the video to support your claims. (students)
3. Discuss the teacher's questioning during the interview. Why did she ask particular questions when she did? (teacher)

The prompts were deliberately broad in an effort see what the PTs in each class section noticed themselves rather than to focus them on just what teacher educators or experienced practicing teachers might think important. All identifiers were removed from the response before coding, including the practicum treatment group in which the PTs were enrolled.

In the first two questions, responses for each question were coded for the mathematics concepts cited by the preservice teachers. Each response that noted a different concept was coded as a separate item. If one PT included both "tens and ones" and "cardinality" in a single response, these were coded as two separate items. After a comprehensive list of items was compiled, similar categories were condensed (Miles and Huberman, 1994). For example, a PT's response that noted "grouping by tens" was included in the "tens and ones" category. The frequency of each code was counted separately for prompt one and both parts of prompt two.

Prompt three asked preservice teachers to discuss why the teacher asked particular questions when she did. Therefore, a domain analysis using *rationale* as the semantic relationship (Spradley, 1980) was conducted to find emerging themes. Starting with the phrase, "x is a reason for doing y," similar rationales were grouped together to look for patterns.

In this instance, the PT is noting that the teacher wanted the student to think in tens. This rationale was expressed by several of the PTs using different questions as their evidence. No a priori codes were used for the analysis of prompt three. Instead, the emergent rationales were determined through iterative cycles of coding and categorizing.

Findings

As anticipated, two-sample T tests showed no difference in mathematics knowledge for teaching between the existing and revised practicum groups (p = 0.5080). This finding was to be expected since both sections were in the methods course with same instructor, same materials and tasks, in the same semester, on the same day. However, differences were clear in the preservice teachers' performance on the video interview assessment for prompts one and two.

Prompt 1. **What mathematical content is the teacher trying to assess?**

Preservice teachers in the revised practicum exhibited a narrowed focus in terms of the mathematical content the teacher in the video was trying to assess. They had fewer responses overall (n=34) and those responses were in fewer categories. All 25 PTs in the group identified tens and ones as a mathematical concept being addressed. Other concepts identified were unitizing (n= 3), and one-to-one correspondence (n=2). One PT each mentioned spatial relationships, counting, more/less, and multiplication/division.

Only 15 out of 25 PTs in the existing practicum identified the goal of the interview as assessing tens and ones. There were more responses overall (n=45) citing a wider range of concepts including addition/subtraction (n=6), counting (n=5), and unitizing (n=4). Cardinality and one-to-one correspondence were noted three times each; more/less and subitizing were mentioned two times each; and multiplication/division, estimation, and spatial relationships were each cited once. Many more preservice teachers cited multiple concepts as the focus of the assessment as if they were listing everything a child would need to know to be successful or anything they had studied in relation to K-2 number sense in the methods course in effort to cover all the bases.

Prompt 2a. **What does the student understand?**

For part one of the second prompt, the existing practicum PTs again offered up more responses (n=62) than those in the revised practicum (n=40). However, overall both groups were similar in terms of what they said the student understood. The explanation for the greater number of responses is that the PTs in the existing practicum listed some concepts that the student did understand, but were much earlier number concepts. These concepts are prerequisites for understanding tens and ones, but were not a focus of the interview (one-to-one correspondence, cardinality, conservation of number). One difference was the number of PTs in each group who thought that Amy understood the equivalence of one ten stick equaling ten single cubes. Six PTs in

the existing practicum claimed Amy understood this concept compared to only one student in the interview-focused practicum.

Question 2b. **What does the student not understand?**

Interestingly, prompt 2b is the only question in which the PTs in the revised practicum had more responses (n=62) than those in the existing practicum (n=35). Approximately equal numbers of PTs in both sections said that Amy did not yet understand addition and subtraction and place value. Almost double the number of PTs in the interview-based practicum also noted (separately from place value) that Amy did not understand grouping by tens/unitizing/number relationships (17 versus 9). Another striking finding was the difference in the number of PTs who asserted that Amy did not yet fully understand comparing/equivalence (11 revised versus 3 existing). The evidence cited by preservice teachers in the revised practicum to support this claim was usually a portion of the interview in which Amy counts the number of cubes in a ten stick and then says that another ten stick of the same length has nine.

In general, PTs in the revised practicum cited more specific instances from the interview to support their claims for prompts one and two. However, this specificity did not carry over into the responses to prompt three about teacher questioning.

Question 3: **Discuss the teacher's questioning during the interview. Why did she ask particular questions when she did?**

Both groups were still unable to explicitly reason about and discuss teacher questioning that fosters student thinking. When asked to discuss the teacher's questioning in the video and to offer rationales for why the teacher asked particular questions when she did, there was no discernable difference in the responses between the existing and revised field experience groups. A few preservice teachers responded with specific evidence to explain possible rationales for the teacher's questioning, but responses overall lacked such specificity in two ways. 1) Some preservice teachers simply summarized the teacher's questioning without any analysis of the rationales. 2) A second group gave general reasoning for the teacher's questioning claiming she asked particular questions to "see what the student understands" or "to see how Amy found the answer."

Discussion

Preservice teachers in the revised practicum group were better able to determine the mathematics goal the interviewer was trying to assess and the content the student in the interview did and did not understand, indicating that the mediated field experience did make a difference.

The revised practicum did not meet the goal of helping preservice teachers become competent in the high-leverage practice of teacher questioning. Their noticing about the teacher's actions and decisions during the interview included vague statements without much evidence to support their answers or without making explicit connections between the evidence offered and the claim made.

Success could be claimed with just the improved focus on determining mathematical goals and analyzing student thinking. Teacher change often begins with a focus on student thinking (Carpenter, et al., 1999) so the fact that the PTs were able to demonstrate a focus on student understanding is encouraging. However, the three points on the instructional triangle (NRC, 2001) do not occur in isolation in the classroom. Teacher actions and decisions are inextricably intertwined with the content and with student thinking. Finding a way for beginning preservice teachers to make sense of the complexity of teaching and to progress with teacher questioning in relation to content and student thinking seems imperative. Both the tasks required in early field experiences and the measures used to assess preservice teachers' intentional use of teacher questioning to facilitate student thinking about mathematics need to be revisited.

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PRESERVICE TEACHERS' UNDERSTANDING OF DECIMAL NUMBERS AND QUANTITIES

Margaret Rathouz
University of Michigan-Dearborn
rathouz@umd.umich.edu

Nesrin Cengiz
University of Michigan-Dearborn
nesrinc@umd.umich.edu

This study explored and characterized representations used by 160 pre-service elementary teachers performing decimal comparison tasks. The PSTs' written explanations for their renaming and comparing strategies were also analyzed. Results indicate that the participant PSTs held misconceptions leading to incorrect decimal notation for amounts such as "16-tenths." The analysis of written explanations showed that PSTs commonly rely on rules when asked to compare decimals. By the end of the course unit, PSTs successfully used appropriate representations and there was improvement in correctness and quality of explanations for renaming and comparing decimal quantities.

Teaching mathematics for understanding requires teachers to understand the mathematics they teach with depth and flexibility (CBMS, 2001; Ball, Hill, & Bass, 2005). Furthermore, to teach in classrooms where there is an expectation for children to justify their mathematical ideas, it is helpful for teachers themselves to learn mathematics in comparable environments. These requirements present challenges for mathematics teacher educators because elementary teachers in the U.S. appear to lack profound mathematical understanding (Ma, 1999). Additionally, many preservice teachers (PSTs) have experienced mathematics as a set of rules and procedures to be memorized, rather than a subject that relies on reasoning (Ball & Bass, 2000).

To address these challenges, a group of researchers at our university conducted an NSF-funded project to develop curriculum materials used in content courses for PSTs. In the study reported on here, we examine PSTs' mathematical conceptions and representations of decimal quantities and their explanations during a unit in the first of these content courses. In particular, we attempt to characterize how their representations, strategies, and explanation change as they engage in cognitively demanding decimal tasks.

Theoretical Framework

The particular focus on meanings and representations of decimal numbers was chosen because decimals represent the convergence of place value and fractions. These are two conceptual fields that have been shown to be difficult for children and their adult teachers (Stacey, et. al., 2001). Limited understanding of base ten relationships in decimal notation and of connections between fractions and decimals leads to patterns of decimal comparison errors that have been well studied (Resnick, et. al,1989; Steinle & Stacey, 2004).

To address these shallow and incomplete understandings, several researchers have suggested activities and tasks to help with making connections among multiple representations (Suh, Johnston, Jamieson, & Mills, 2008). Representations that embody the base ten structure of the decimal number system, such as 10-by-10 grids, have been proposed to help learners visualize the order of magnitude represented by place value positions (Martinie & Bay-Williams, 2003).

Introducing these representations imbedded in tasks that require a high level of cognitive demand can further support student learning. For example, asking students to solve problems in more than one way, to make connections among solution methods, and to explain their thinking, all contribute to the cognitive demand of the task. Maintaining that level during classroom instruction is critical in developing true understanding. Building on students' prior knowledge, allowing sufficient time to explore, and pressing students to justify their reasoning are all factors associated with the maintenance of high-cognitive demand tasks (Stein, Smith, Henningsen & Silver, 2009). Further, reflecting on and reasoning about others' mathematical thinking focuses our students (PSTs) on part of the real work of teaching.

Methodology

The data for this study were gathered from the first content course for preservice elementary and middle school teachers. The data included pre- and post-tests on PSTs' knowledge of decimals, PSTs' written work (homework assignment and exams), videotapes of lessons from the decimals unit. In this report, we discuss selected findings from the pre- and post-tests.

In the content courses, lessons are 100 minutes long and PSTs usually work on two or three tasks during one session. These tasks are designed to provide PSTs with opportunities to explore and make connections among important mathematical ideas. The PSTs typically work on the tasks first individually, then share their thinking in small groups. Finally there is a discussion where the solutions are shared and synthesized as a whole group. The focus of in-class discussions is not only how the problems are solved, but, more importantly, why the solutions make sense. The expectation of providing justifications for solutions and mathematical claims challenges PTSs' thinking.

The unit on decimals is designed to support PSTs' developing knowledge about meanings of decimals, representations of decimals, and connections among these meanings and representations. See Table 1 for selected decimals tasks. Throughout the unit PSTs are encouraged to use representations (such as 10-by-10 grids) to support their justifications.

Table 1.

Selected decimals tasks from coursework.

Renaming 0.1	Rename this number in more than one way. You could use numbers, words, drawings, and/or contexts.
Representing fractions as decimals	Find four different ways to show 1/4 on a 10 x 10 grid by shading in that portion of the grid. Try to choose ways that allow you to clearly see the result is 1/4 without requiring that a person count every shaded box.
Analyzing students' re-namings	Several students were discussing different ways to name 245-hundredths. In evaluating the following students' claims, first create your own representation of 245-hundredths using 10 x 10 grids. Bradley said that 245-hundredths could be represented as $\frac{2450}{1000}$. Use both words and a diagram to validate Bradley's claim. Amira said that she could write 245-hundredths as 0.245. Use both words and a diagram to persuade Amira that these two expressions are <u>not</u> equivalent.

We collected data throughout five semesters (Winter 2008 – Fall 2010) from 160 PSTs enrolled in the first content course. The pre-test was given before the Decimals Unit began and the post-test was given at the end of the unit. In this paper, we focus on only one pair of corresponding items about comparing decimals from the pre-test and the post-test. The items are listed in Table 2 below. The task required PSTs to compare two amounts that are written in numerals and words and to provide explanations for their comparisons. To compare the given amounts, it was necessary to rename the quantities. This aspect of the item revealed PSTs understanding of renaming and comparing decimals and their use of different representations.

Table 2.

Items about comparing decimals.

Pre-test Item	Post-test Item
Which one is bigger: **16-tenths** or **134-hundredths**? Show all work and briefly explain how you decided which is larger.	Which one is bigger: **18-tenths** or **172-hundredths**? Show all work and briefly explain how you decided which is larger.

The analysis of PSTs' written work and their representations was an iterative process. First, we categorized the types of representations that were used: Symbolic (fractions, decimals, whole numbers, and percent); verbal (place value names and fraction language); everyday life context (money, etc.); and diagrams (number line, area models –fraction strips, circles, rectangular, 10-by-10 grid, and discrete models). The correctness of the representations was captured during that analysis.

Next, PSTs' strategies of renaming and comparing decimals were analyzed. The categories of strategies for renaming decimals included (a) finding a common denominator; (b) making a whole with some leftover; and (c) positioning the right-most digit of the numeral in the named place value position (i.e. for 134 hundredths, the "4" would be placed in the hundredths place); (d) using division. The strategies (including both correct and incorrect ones) for comparing decimals were (a) using common denominators or same length decimals; (b) using a benchmark; (c) comparing matching place value positions; (d) comparing the denominators or place value names/positions (i.e., tenths are larger than hundredths); (e) comparing the numerators or digits; (f) shorter is larger (e.g., 0.6 is closer to the decimal point); (g) longer is larger; (h) erroneous place value comparison; and others.

PSTs' explanations for both renaming and comparing decimals were evaluated using the following levels: explicit and valid explanation (3); partial explanation (2); description of the process or rule without explaining why (1); no explanation, only computation or representation (0); and confusing/incorrect (-1).

Findings

Type and Accuracy of Representations
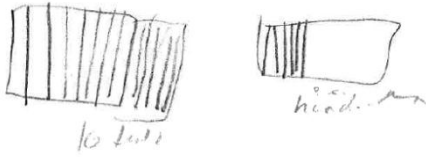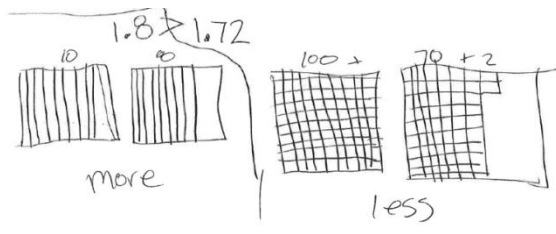
The analysis of PSTs' responses on the chosen items from the pre-test and the post-test indicated that the most commonly used representation was decimal notation. Nearly half (46%) of the PSTs used decimal notation to represent 16-tenths and 134-hundredths on the pre-test. Of those who used decimal notation on the pre-test, 78% used it incorrectly (e.g., most common representations of 16-tenths and 134-hundredths were .16 and .134, respectively). Using decimal notation to represent given numbers was also common on the post-test, with 56% of the PSTs using this representation and 83% of those reporting the correct decimals, a highly significant increase in the proportion of correct responses from pre-test to post-test, $p < 0.001$, using a 2-proportion Z-test, $z = 9.97$. It was also common to see the given amounts represented as fractions on both pre- and post- tests. The PSTs seemed to be comfortable translating to fractional forms, such as 16-tenths to $\frac{16}{10}$. Consequently, most of these representations were correct.

The kinds of diagrammatic representations that PSTs used included number lines, discrete models, and area models such as circles, rectangles, and 10-by-10 grids. Diagrams were rarely seen on the pre-test, but of those, 10-by-10 grids were the most common. Only four PSTs generated or referred to 10-by-10 grids on the pre-test, and of these, only one was correct. Both the use and the

correctness of 10-by-10 grids increased from pre- to post-test. Of the thirty-three (21%) PSTs who used 10-by-10 grids on the post-test to represent the given amounts, 82% did so correctly. Fisher's exact test for small samples was used to show the result was significant, *p = 0.012*. See Table 3 for sample pre- and post-test responses.

Table 3.

Sample PST responses for using 10-by-10 grids.

An incorrect response from the pre-test	A correct response from the post-test
16-tenths vs. 134-hundredths	18-tenths vs. 172-hundredths

Strategies and Explanations

The PSTs used a number of different strategies to rename and compare the given amounts. Several of the strategies suggested that PSTs held misconceptions about decimal quantities. We characterized three of the correct renaming strategies that we observed and labeled them (a) finding common denominators or same-length decimals and comparing numerators; (b) making a whole and comparing the "left overs"; and (c) positioning the right-most digit of the numeral in the named place value position and comparing like place values. In this paper, we only discuss findings about strategy (a) because this was most commonly used and because there was a dramatic increase in both the number of PSTs using this strategy correctly and the sophistication of their explanations.

On the pre-test 18% of the PSTs used this method, which involved renaming quantities as fractions or decimals and then comparing the numerators or the numeric digits of the decimal numbers as if they were whole numbers. For example, some renamed 16-tenths as 160-hundredths whereas others renamed 134-hundredths as 13-tenths and 4-hundredths. This strategy was rarely explained or justified by PSTs on the pre-test, but more frequently was simply described. Of the 18% of responses that involved this strategy on the pre-test, 73% were scored 0 or 1 point, indicating a computation only or a description of a rule the student followed. For example, a 0-point response from one PST was "$\frac{16}{10} \times \frac{10}{10} = \frac{160}{100}, \frac{160}{100} > \frac{134}{100}$." Typical 1-point explanations were, "16

tenths is bigger than 134 hundredths. I decided that because when they have the same number of places, 160 is bigger than 134." And, "Adding a zero to the end will make 18-tenths 180-hundredths without changing the amount."

The overall use of common denominators increased on the post-test as well as their correctness, completeness, and sophistication of explanations. On the post-test, 49% of the PSTs used this method, representing a highly significant increase, $z = 6.23, p<0.001$, in use of this strategy from the pre-test. Of these, 23% scored 2 or 3 points, indicating that these responses included partial or full explanation for equivalence and comparison of decimals. Several explanations focused on creating equal-sized pieces. For example, one student wrote, "If we rename the 8/10 left over into 10 times smaller pieces, there would be 80/100 pieces. Compared to 72/100 pieces, there are more with 80/100." Some PSTs referred to 10-by-10 grids: "180 hundredths vs. 172 hundredths. 18 columns with 10 boxes in each column. That gives you 180 boxes shaded, which is 180 hundredths because the columns are tenths and the boxes in the tenths are hundredths. 180-hundredths is more than 172-hundredths." There were a few PSTs who made reference to everyday life contexts, such as currency: "I looked at it as 18 dimes, because a tenth of a dollar is a dime so if I had 18 dimes I have $1.80. If I had 172 hundredths that would be 172 pennies because a penny is a hundredth of a $1. So 1.80 is more than 1.72."

Misconceptions

Careful examination of strategies and explanations revealed misconceptions held by the PSTs in our study. The types of misconceptions, such as "using tens and tenths interchangeably," "longer is larger," "tenths are larger than hundredths," and "closer to the decimal point is larger" are documented in the literature and were most frequently observed on the pre-test. Table 4 provides examples of explanations characteristic of these types of misconceptions.

Table 4.

Examples of explanations for common misconceptions.

Misconception/Misunderstanding	Explanation	Frequency
Tenths are Larger than Hundredths	"When you have 16 tenths, it's bigger because each piece is bigger. With hundredths, you are basically chopping the tenths up into hundredths."	Pre: 18 Post: 8
Closer to the Decimal Point is Larger	"16 tenths is bigger because tenths is closer to the decimal, meaning it's closer to being a whole number."	Pre: 18 Post: 5
"Tens" for "tenths"	"16 tenths can be written 160; ten sets of 16 are being added. 134 hundredths is 13,400. So 134 hundredths is larger."	Pre: 21 Post: 0

Conclusion

Our students, prospective elementary teachers, seem to be fairly typical in terms of their over-reliance on rules and procedures they may not understand or remember. In comparing decimal quantities, most represented the given amounts using symbolic notation and did so incorrectly. Our data support prior studies that suggest adult learners exhibit some of the same decimal misconceptions that children have.

Future teachers struggle to understand the meanings of decimal numbers and to use those meanings to solve problems of equivalence and comparison. During our instructional unit on decimals, the introduction of representations such as 10-by-10 grids appears to support their reasoning about the relative sizes of decimal quantities. They begin to move flexibly between fraction and decimal notation. Further, challenging them on written assignments to analyze others' solutions, both correct and incorrect, and pressing PSTs for justifications during whole group discussions, encourages students to explain their reasoning and provide convincing arguments to others. Explaining why methods work allows these future teachers to examine and reflect on their own, often incorrect, procedural knowledge and provides them with an opportunity to practice precise mathematical language that they will use later in teaching.

Our research is part of a wider effort by the community of mathematics educators to define the mathematical understandings future teachers need to develop. Analyzing how PST mathematical thinking changes after experiencing our curriculum materials and classroom instruction is allowing us to create and refine course tasks and their implementation to optimize the mathematical learning of PSTs.

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A SYSTEMIC APPROACH TO TEACHING MATHEMATICS FOR SOCIAL JUSTICE WITH PRESERVICE TEACHERS

Summer Bateiha
Western Kentucky University
summer.bateiha@wku.edu

Melanie Autin
Western Kentucky University
melanie.autin@wku.edu

Hope Marchionda
Western Kentucky University
hope.marchionda@wku.edu

Teaching social justice mathematics has become increasingly more popular in recent decades. However, not many instances of successful teaching experiences of this sort exist in the literature. This study examines how one instructor infused ideas of social concern in a mathematics content course she taught for preservice elementary teachers. Upon careful examination, it appeared that teaching in this way is most successful when the evolution of the course is thought of as an emergent, dynamic, complex system.

In recent decades, concern for critical global issues have been on the rise (Bender, Burns, Burns, & Guggenheim, 2006; D'Ambrosio, 2007). Issues such as poverty, healthcare, and the effects of consumerism on our society have come to concern educators. Efforts to address these issues and encourage social action for and through mathematics have emerged in the form of a social justice approach to teaching mathematics (e.g., Gutstein & Peterson, 2006). Within this form of pedagogy, mathematics is used to reveal the effects of injustice through mathematics and is turned into a "tool to understand and potentially change the world" (Gutstein & Peterson, 2006, p. 2). The basic premise is that if students can understand mathematics sufficiently enough for "*reading* the world" through mathematics, they can use this knowledge to gain power and authority in their own lives (Gutstein, 2003) by recognizing injustice so that they may envision alternatives to the destructive aspects of society. Further, teaching in this way can stimulate mathematical interest by connecting it in more meaningful ways to a world outside of the classroom.

Although mathematics educators have been exploring the idea of teaching mathematics in conjunction with social issues for some time now (e.g. Powell & Frankenstein, 1997; Gutstein & Peterson, 2006; Spielman, 2009), not many studies of how to create successful experiences exist in the literature. Therefore, this paper explores how one educator instigated the construction of a social justice mathematics content course that was well received by a group of elementary preservice teachers. In this paper we examine two components of the course that seemed to contribute to preservice teachers' understanding and appreciation of both mathematics and social issues.

Theoretical Lens

The world we live in is increasingly being understood as a series of complex systems and subsystems (Capra, 1996). Every field from biology to economics to social studies has found structures that are inexplicable when reduced to their individual parts. For example, biologists have found that they are "unable to explain the self-preservation of the animal organism by recourse to the physical laws governing the behavior of its atoms and molecules" (Laszlo, 1996, p. 8). Social scientists have begun to understand that often times, beings or large groups of things have their own personalities that can remain intact or shift very slowly, even when individual members change or are wiped out. A football team is an entity which will replace members throughout the years but may continue to maintain its characteristics – "their tactics and techniques, their fighting spirit, and so on" (Laszlo, p. 5). Such structures cannot be reduced to the characteristics of the individual parts but nevertheless exhibit unique characteristics as wholes (Briggs & Peat, 1989).

Viewing the make-up of the universe as a series of systems carries many implications about finding solutions to the problems our culture has come to possess. Scholars, such as Fritjof Capra, have begun to propose solutions that would have previously seemed incomprehensible and illogical. For example, he (Capra, 1996) explains that an interconnected, paradoxical view can eliminate problems such as overpopulation. He proposes that "stabilizing world population will be possible only when poverty is reduced worldwide" (p. 3). Rather than thinking in terms of hierarchy and separation, solutions may be found in horizontal and interdependent ways. Human beings begin to be understood as systems, living within a culture or larger system, which is part of a world or an even larger system, and so forth. This systemic view of our universe leads to the implication that all systems must be seen in relation to one another and survival of each means sustenance for all.

Further, a systemic approach to life carries with it implications for education--all education-- even in the field of mathematics. From this perspective, the mathematics classroom and all its participants, students, subject, environment, and teacher alike, can be viewed as complex systems that interact to form a whole that cannot be separated and understood or analyzed by its parts but rather exists in the interactions between and among them. Because of the dynamic nature of this system, no two teachers can be given the same set of procedures for teaching a course and the same results emerge. Therefore, the evolution of this social justice mathematics cannot be imposed on others for the purposes of creating the same results; however, what it may provide is some insight

into how one educator began transforming a prescribed mathematics curriculum into a broader, systemic social justice mathematics experience.

This study explores how a social justice classroom emerged from a systemic lens. Although the authors attempt to analyze components in order to share the story, the success of the evolution of the course did not materialize from having been treated as a set of predefined, prescribed procedures. The course turned out the way it did because it evolved naturally, based on the interactions and reactions of the participants involved.

Methodology

Nineteen female preservice elementary teachers and the instructor of the course constituted the participants in this study. These preservice teachers were enrolled in the first content course of a two-course sequence at a community college in the Southwestern region of the United States. Seven of the students were non-native speakers of English, and several of the participants were nontraditional students. The class met for three hours, once a week, for fifteen weeks. Students were given activities to work on throughout the week, all of which were to be attempted before ever being discussed in class. The instructor of the course was also a participant and a researcher in the study. Similar to several of her students, she was a student (a doctoral student), a female, a mother, and also a second language speaker of English.

The study was a qualitative one that combined case study (Stake, 1995) and practitioner-research (Anderson, Herr, & Nihlen, 1994) designs. Data for the study were collected using journal reflections from the instructor and students, student work, and audio/video recordings of the class. A data analysis spiral was used to analyze data (Cresswell, 2007). Data were continuously cycled back through both during the semester and after its conclusion.

Findings

The preservice teachers in this course described several important aspects of the course that aided in their development of a conceptual understanding and interest in both mathematics and social issues, but the most popular aspects were the open dialogue encountered, and the social issues chosen. The initial interactions between teacher and students are the focus of this paper.

Open Dialogue

Most traditional mathematics classrooms follow a strict regimen of lecture, individual solving of homework problems, and feedback that is limited to grading students' work. In this classroom, we consider how breaking down the dichotomous nature of teaching and learning provided a space for

open dialogue which prompted students' to construct and negotiate meaning of mathematics by giving and receiving instantaneous feedback (Davis, 1997). Developing a community in the classroom where students could teach and interact openly with one another and the instructor broke down the traditional power structures that exist between authority (teacher) and novice (student). This open dialogue created space for a caring environment where authentic respect could emerge and teacher as well as student could mutually and actively engage in the construction of meaning. The breakdown of traditional hierarchies between individuals was described by the instructor of the course as one of the main social issues she wished to incorporate in order to create a more meaningful learning environment and implicitly illustrate the limiting effects of traditional power structures in society.

The breakdown of the teacher-learner power structure began on the first day. After discussing the syllabus and telling the students a little about herself, the instructor asked each student to share something personal with the class. The instructor wanted students "to begin the process of speaking with each other by talking about something familiar, rather than beginning with a discussion about mathematics, which I anticipated might be intimidating…" Although she wrote in her reflective journal that she sensed that students were "tense about and nervous with the idea of talking on the first day," she wanted "to break the ice somehow." Further, she wanted "to model the communication process." She did this by reiterating many of the statements students made and connecting them to the accounts of others. She immediately began using students' names, "as a way to emphasize their value as individuals rather than random students." In her reflective journal, the instructor wrote:

Two students, Erica and Wanda, told the class they were pregnant, and another, Alex, humorously said she had recently gotten married and was trying to get pregnant. I joked and said, "Maybe you should sit next to Erica and Wanda. It might be contagious." Several students laughed at this statement. I wanted students to see that I was listening to them and interacting with them. I also wanted them to see that I recognized commonalities between them, and this was not going to be a serious place where only formal interactions between students and teacher exist. I wanted them to relax.

Next, the instructor randomly placed students in small groups and asked them to discuss an introductory mathematical task. This initial group work was a way to engage the students in a negotiation of the social norms for the class. "I wanted students to begin accepting each other as

having equal value. The goal of this assignment was to introduce students to the idea that they would all be active participants who are respected and heard in this course." According to Noddings (1992) "[s]tudents will not succeed academically if they are not cared about…Dialogue taking place while learning in communion connects children [or adults, in this case] to each other, and it provides knowledge about each other that forms a foundation for caring" (p. 23). Setting the stage in this way would carry out through the remainder of the semester. This was the mathematical task she gave them:

> Find the next three terms in the following sequence: 7, 11, 15, 19, 23,…(This is called an arithmetic sequence.)
>
> Find the next three terms in the following sequence: 3, 6, 12, 24, 48,…(This is called a geometric sequence.)
>
> What do you think is the difference between an arithmetic sequence and a geometric sequence?

After they discussed the task in groups, she invited each group to share what they found. She also asked students whether or not they agreed with other groups' ideas, as they emerged. We illustrate how this transpired with a reflection the instructor wrote on a whole-class discussion of the task above.

> Mary volunteered to show the class how her group solved the first question. She wrote on the board:

$$7, 11, 15, 19, 23, 27, 31$$

> She explained that her group added four to generate the next three terms of the sequence. I reiterated Mary's statement, and asked the class if anyone had done the problem any differently. No one interceded. Then, I asked Kyran if she minded illustrating to the class how her group solved the second problem. Although she seemed hesitant, she came to the board and wrote:

$$\overset{\times 2 \quad \times 2 \quad \times 2 \quad \times 2}{3, \ 6, \ 12, \ 24, \ 48,}$$

> She explained that her group noticed that each term was multiplied by two to generate the next term of the sequence. After Kyran explained that the next three terms would be 96, 192, 384, I asked the class if anyone had anything different. Megan interjected that her

group "added the double." I asked her to show the class what she meant. Megan wrote on the board:

$$\begin{array}{cccc} \times 2 & \times 2 & \times 2 & \times 2 \\ 3, \ 6, \ 12, \ 24, \ 48, \\ +3 \ +6 \ +12 \ +24 \ +48 \end{array}$$

She explained that her group generated the numbers in the sequence by doubling the number they were adding each time. They began with three and added it's double, six, the next time, repeating this pattern until they generated three new terms. Megan's method surprised me and prompted me to ask the students what they thought the difference between an arithmetic and geometric sequence is. Megan said her group had decided that "an arithmetic sequence is generated by addition and a geometric is generated by multiplication." Kyran, on the other hand, said her group had written "an arithmetic is generated by adding or subtracting and a geometric is generated by multiplying or dividing." I asked the class if there was a difference. Another student explained there was not and wrote the following on the board:

$$3 - 2 = 3 + (-2) \qquad\qquad 4 \div 2 = 4 \text{ x } \tfrac{1}{2}$$

Although some students seemed troubled by these statements, the conversation took another direction. Angel asked a question:

> Angel: Megan added up there, so can a geometric sequence be arithmetic?
> [Angel was referring to Megan's assertion that "an arithmetic sequence is generated by addition and a geometric is generated by multiplication."]

Angel's question made sense to me. She was confused by the other students' definition of a geometric sequence dealing with multiplication and division alone when Megan's method for generating new terms included addition.

Several students began participating in the conversation at this point. One student suggested that both sequences could be arithmetic. Then another noticed that in the first sequence the same number is added and in the second, a different number is added. Andrea told the class that an "arithmetic is adding the same number over and a geometric is multiplying the same number over and over, even if you are adding those numbers." Kim called this number a "constant." This led Angel to conclude that the two sequences are different. I asked April to tell the class what she thought the difference was between an arithmetic and a geometric sequence. After she spoke, in my own words, I retold the

incident and asked the class to help me write the definitions for the two sequences on the board.

At this point in the class, the students were able to agree upon correct definitions for the two types of sequences. Although the instructor could have easily surpassed all the questioning and debating that occurred and could have simply told her students what arithmetic and geometric sequences are and how to generate terms for each, the discourse that came to light was much more than just reaching conclusions about how to generate terms for arithmetic and geometric sequences. In this discussion, students had to communicate with classmates to make sense of the meanings other students had constructed. Angel heard Megan say that an arithmetic sequence uses addition and a geometric uses multiplication, but she saw Megan add in the geometric sequence. This led Angel to ask whether or not a geometric sequence could be the same as an arithmetic one. However, by listening to the discussion, Angel understood that adding could be one way to generate terms of a geometric sequence but that did not mean it could be characterized as an arithmetic sequence.

Through their discussions on the first day of class, students exchanged ideas with one another and discovered that they could construct knowledge without relying on a teacher to tell them the correct answers. This became a catalyst for developing mutual respect for one another's ideas. Dialogue began to occur between and among students and instructor (Davis, 1997), and students began to have an equal voice in the classroom. They learned to display their knowledge by constructing it within the process of open communication, and they gradually began to perceive the instructor as a participant and facilitator in the process of formulating ideas and meaning.

Social Issues from Student Interest

Although the episode above illuminated the open dialogue that instigated the breakdown of power structures (a social issue the instructor set out to attend to), it did not explicitly address mathematics content problems embedded within a social context (another of the instructor's objectives). The process of utilizing social issues to discuss mathematics became a major component of the course; however, it materialized gradually. Aware that a delicate balance exists between dissonance and affective safety, the instructor of the course indicated that she wanted to approach the introduction of social issues in educative rather than "miseducative" ways (Dewey, 1938). Dewey described the need for disequilibrium for the attainment of new knowledge; however, he also expressed that too much dissonance is dangerous and can cause what he referred

to as "miseducative experiences," or a hindrance of the creation of new ideas. Therefore, a difficulty the instructor described encountering was negotiating where that balance lay when discussing critical social issues with mathematics. She decided to utilize student interest as the driving force for the social issue lessons.

The initial social issue was chosen by the instructor; however, it became a catalyst for discussing society in the class and prompted students to share concerns they had about society. The instructor then used these interests and concerns to develop the lessons that included social issues. In the first set of activities, the instructor included four tasks that addressed poverty, one of which asked students to create a monthly budget for a family of three living in poverty. Students had to research government aid programs in order to figure out how to spend such a limited amount of money over a thirty day period. She then asked students to reflect on this process. They discussed students' budgets during the third week of class. For some students, completing this task was not an abstract assignment, since they lived under similar circumstances. However, for others it was difficult to imagine such a financial situation. Every student wrote about how a family in this financial situation is, as one student put it, "living in a very dangerous place." For example:

> I had some difficulty because most of these expenses I do not have myself so it was hard for me to make sure all the basics were covered and I'm sure many necessities were left out or underestimated which is bothersome because there was already many areas such as insurance and rent that may have already been estimated a little lower than possible. To me, [this] puts quite a strain on day to day living...

Student concerns such as this one's emerged during the whole-class discussion of this assignment. This prompted some to share their personal experiences with living under strict financial guidelines. In her reflective journal, the instructor wrote about one student who told the class that she could not afford healthcare because she battled diabetes, and insurance companies would not accept her with a "preexisting condition." This story instigated a discussion about whether healthcare reform was necessary or not and students began expressing their opinions about the issue.

Based on student interest, the next set of activities concerned the healthcare reform debate of the time. In an attempt to create a connection between proportional reasoning (a major mathematical objective of the course) and understanding major current issues, the instructor asked the students to watch President Obama's healthcare speech (In Full, 2009) and the Republican response

(Republican Response, 2009). This preceded a mathematical debate over the healthcare reform bill (which included the "Public Option") of the time. Students were asked to research the mathematics used for or against the bill. They were asked to use and explain the mathematics they found (with citations) to convince their classmates of their points of view. The mathematics students brought in included mostly decimals and percentages, but there were fractions and graphs as well. For example, one student found that the "United States had not contained costs as effectively as nations with broader public coverage. As the *OECD* [Organization for Economic Co-operation and Development] *Observer* notes: 'U.S. health expenditure grew 2.3 times faster than GDP [Gross Domestic Product], rising from 13% in 1997 to 14.6% in 2002. Across other OECD countries, health expenditure outpaced economic growth by 1.7 times.'" After sharing this example, this student had to explain what these numbers meant, in context, when looking at GDP.

Students were divided into two groups, Republicans and Democrats. Students were intentionally placed so that each side had students who did and did not identify with that political party. The groups had to use mathematics to make arguments and convince each other of their political party's perspective. During the debate students discussed mathematics that was not explicitly addressed in the assignment. For example, one group brought in a set of graphs that depicted "Annual Small Business Profits Lost Due to Healthcare Costs" and "Estimated Family Income with and without Health Care Reform." They had to explain what the graphs meant and how they supported their argument.

Creating activities that were based on student interest in social issues seemed to engage students in the tasks and impact their understanding of both mathematics and social issues. Every student described learning something from the lesson, and more than half wrote about enjoying engaging in this debate. When they discussed what they learned, students connected mathematics to a relevant issue in their lives. Just as students negotiated meaning with the sequence task, they were asked to negotiate the meaning of the mathematics they were using to make their arguments about healthcare reform. At no time during the debate did the teacher indicate her own perspective on healthcare reform, allowing students to make decisions about social issues without her interference.

The instructor of the course used this technique of revolving mathematics lessons around student interests throughout the semester. Students as well as instructor became consumers of information by understanding mathematics and mathematical thinkers by consuming social issues.

Learning what social issues interested students allowed the instructor to become a more effective teacher.

Implications

The introduction to this course set a foundation for the community atmosphere of the class. It created a comfortable environment where students could engage in open dialogue and begin to perceive themselves as teachers as well as learners. This provided opportunities for students to be heard, where they could explicate their thinking through interactions with the teacher and other students. The care and respect students encountered enhanced mathematical understanding for most of the students and dissolved many students' misconceptions about who holds the knowledge (Davis, 1997). One student seemed to sum it up when, after several classroom meetings, she wrote,

> I think group work is excellent. It really helps people understand with every type of thinking. I also think it is great to have interaction with the teacher. It shows the teacher really cares and it helps with the learning process.

Further, the environment that emerged prompted an openness and communication that invited students to participate in not only talking about mathematics in meaningful ways, but also engaging in lessons that addressed social issues. As a part of valuing students' voices (Davis, 1996) and with the recognition of the delicate balance that exists between dissonance and affective safety (Piaget, 1972), the instructor utilized the interests of students in developing these lessons. The result was an authentic involvement in the learning process. Although not discussed in this paper, the evidence existed in not only the amount of communication and mathematical development that occurred during the lessons, but also the fact that several students had definite perspectives on many of the issues when the explorations began, and those changed during the span of the assignments. Learning what students were interested in helped facilitate mathematical as well as social development.

It was the complex systemic approach to teaching this course that made it a success. It was the recognition that humans are a part of a community of learners when they are in the classroom and the importance of treating them as such. The classroom is not separate from an outside world, and the issues that concern students in their surroundings can engage them within the confines of a classroom. The instructor understood the importance of integrating mathematics and social issues in a gradual manner so that the students would feel safe, not fearful, angry, or resentful. It was the understanding that teaching in this way can only work if it is emergent, dynamic, and based on the

reactions and interactions of the participants involved that made this semester a successful attempt at teaching mathematics for social justice.

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EXAMINING THE ROLE OF INSTRUCTION IN MATHEMATICS TO PREPARE PRESERVICE TEACHERS FOR MATHEMATICS INSTRUCTION IN GRADES 7-9

Travis A. Olson
University of Nevada, Las Vegas
travis.olson@unlv.edu

Melfried Olson
University of Hawaii at Manoa
melfried@hawaii.edu

In this paper we identify, through student self-response survey data, when preservice secondary mathematics teachers' (PSMTs) report receiving instruction in college-level mathematics classes that specifically relate to mathematics content taught in Grades 7-9. Instruction in mathematics courses for PSMTs should emphasize the connection between the mathematics being learned by PSMTs and content they will teach (Wu, 2011). In this paper, data are presented from a survey given to PSMTs to determine whether in their mathematics content courses the PSMTs encounter various concepts for teaching Grades 7-9 mathematics.

There is a growing body of work on the importance of mathematics content and pedagogical knowledge of elementary and secondary teachers (see, e.g., Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005). In particular, PSMTs' struggle to explain, via modeling and representations, solutions to contextualized fraction and ratio problems (Olson & Olson, 2011a; Olson & Olson, 2011b; Sjostrom, Olson, & Olson, 2010). It is thusly important to explore when in their mathematical preparation PSMTs experience learning mathematics in the context of teaching students in Grades 7-9[2].

The question arises as to when PSMTs should receive the preparation to deeply understand, and teach, content in middle-grades mathematics. With respect to the need to deeply understand K-12 mathematics, Wu (2011, Fall) notes that, "Because of the teacher preparation programs' failure to teach content knowledge relevant to K-12 classrooms, the vast majority of preservice teachers do not acquire a correct understanding of K-12 mathematics while in college" (p. 9).

For more than a decade, there has been increased interest from the mathematics community to address this issue. The Conference Board of Mathematical Sciences (2001), in concert with the Mathematical Association of America, developed the *Mathematical Education of Teachers* (CBMS MET) report. This report, to help address the need for specialized courses and materials, articulates a framework for mathematics content courses for preservice teachers. The authors of the CBMS MET report indicated that teacher preparation programs for preservice high school mathematics teachers were not focusing on the variety of connections among algebra geometry, and functions

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[2] In this paper, for variety, the grade-level span, Grades 7-9, is interchangeably referred to as middle-grades or early-secondary grades. The latter distinction is due to secondary teacher certification often spanning Grades 7-12.

and the programs did not focus on conceptual development of PSMTs' understandings of data analysis or discrete mathematics concepts.

Consequently, to better understand where PSMTs encounter college-level conceptual development of their own understandings pertaining to middle-grades mathematics concepts, PSMTs were surveyed regarding content connections between college-level mathematics courses and Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative, 2010). Specifically, the survey was designed to, in part, address the following research question: Where in their college mathematics coursework for certification do preservice secondary mathematics teachers (Grades 7-12) experience learning mathematics content that is specifically related to topics they will likely teach, as defined by certain CCSSM associated with Grades 7-9?

Methodology

PSMTs who were enrolled in a course in teaching methods in secondary mathematics[3] were surveyed[4]. As such, the PSMTs surveyed were, at the time, seeking certification in teaching secondary mathematics[5]. Methods courses were identified at nine institutions of higher education[6] (IHE), and the course instructors were sent an electronic survey link and asked to provide the link to their students. In total, 23 PSMTs at three IHE fully responded to the survey[7].

The first, and successive pages, of the survey displayed the following statements and prompts:

This survey asks where in your preservice college mathematics coursework the instructor or professor emphasized that the content being taught was specifically related to content you might teach as a secondary mathematics teacher.

Because names of courses vary by institution, we have listed courses either by a course name found at certain institutions or by content areas that would be included in them. Please indicate (by clicking for a checkmark) in which course or courses you were taught how the mathematics in the course was related to these concepts into your future as a secondary mathematics teacher.

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[3] Hereafter these courses are referred to as "methods courses" or a "methods course," and unless otherwise noted, are specifically intended for preservice secondary mathematics teachers.

[4] None of the courses surveyed were taught by either of the authors.

[5] For the purposes of this paper, secondary certification is assumed to at least span Grades 7-12.

[6] There was only one methods course identified at each IHE.

[7] It is unclear, due to the method by which the survey was sent, how many PSMTs were provided the link to the electronic survey. As such, there is no reliable way to measure response rates.

That is, was there intentional instruction related to how you might connect the mathematical content of a particular course to how that knowledge is situated in the secondary mathematics curriculum.

On every page of the survey, students were provided the following prompt as related to each standards from CCSSM (see Table 1 for the list of CCSSM standards included in the survey):

In which of the mathematics courses were you taught how the content of the course related to your future as a mathematics teacher who would teach children about analyzing proportional relationships and using them to solve real-world and mathematical problems[8]? Check any that apply.

Mathematics courses listed in programs of study of PSMTs at four IHE in four different states in the United States were identified. Courses that were similar in title and content were grouped under general course titles that represent a comprehensive list of courses reflecting program requirements among the IHE. That is, each required course, and most electives of each IHE is included in one of the categories. The following "course" options were included survey:

Calculus or Advanced Calculus; Discrete Math or Computer Science; Linear or Matrix Algebra; Abstract Algebra or Group Theory; Euclidean, Non-Euclidean, or Projective Geometry; Probability, Combinatorics or Statistics; Sets and Logic; Number Theory; Theory of Equations; History of Mathematics; Numerical Analysis; Real or Complex Analysis; None of the Listed Courses; and Other.

If respondents clicked "Other", they were prompted to provide a written response. Any number of multiple choices of the listed courses could be chosen per prompt.

Findings

Counts were recorded for the number of times a course was identified by a PSMT for a particular CCSSM. Of the 23 participants who completed the survey, 13 were Female, 9 Male, and one did not identify a gender.

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[8] The statement in bolded text specifically relates to the CCSSM standards in Table 1, the standards around which we focused the survey.

Table 1.

CCSSM Standards in the Survey, and Associated Domains and Grade Level Codes

Standard	Domain
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Ratios and Proportional Relationships (7.RP)
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	The Number System (7.NS)
Use properties of operations to generate equivalent expressions.	Expressions and Equations (7.EE1)
Solve real-live and mathematical problems using numerical and algebraic expressions and equations.	Expressions and Equations (7.EE2)
Draw, construct, and describe geometrical figures and describe the relationships between them.	Geometry (7.G1)
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Geometry (7.G2)
Use random sampling to draw inferences about a population.	Statistics and Probability (7.SP1)
Draw informal comparative inferences about two populations.	Statistics and Probability (7.SP2)
Investigate chance processes and develop, use, and evaluate probability models.	Statistics and Probability (7.SP3)
Know that there are numbers that are not rational, and approximate them by rational numbers.	The Number System (8.NS)
Work with radicals and integer exponents.	Expressions and Equations (8.EE1)
Understand the connections between proportional relationships, lines, and linear equations.	Expressions and Equations (8.EE2)
Analyze and solve linear equations and pairs of simultaneous linear equations.	Expressions and Equations (8.EE3)
Define, evaluate, and compare functions.	Functions (8.F1)
Use functions to model relationships between quantities.	Functions (8.F2)
Understand congruence and similarity using physical models, transparencies, or geometry software.	Geometry (8.G1)
Understand and apply the Pythagorean Theorem.	Geometry (8.G2)
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Geometry (8.G3)
Investigate patterns of association in bivariate data.	Statistics and Probability (8.SP)
Extend the properties of exponents to rational exponents.	The Real Number System (NRN1)
Use properties of rational and irrational numbers.	The Real Number System (NRN2)
Reason quantitatively and use units to solve problems.	Quantities (NQ)

The data in Table 2 indicate the year of their program in which they were currently enrolled. The data in Table 3 indicate the type of certification program in which the PSMTs were currently enrolled.

Table 2.

Current Year in Certification Program

Year in Certification Program	Number of Respondents
Freshman	1
Sophomore	0
Junior	1
Senior	16
First Year Alternative	3
Second Year Alternative	1
Third Year Alternative	0
No Answer	1

Table 3.

Type of Certification Program

Type of Certification Program	Number of Respondents
Undergraduate	18
Graduate leading to certification and masters degree	2
Alternative Route (non-degree leading)	2
No response	1

The data in Table 4 represent the counts of each mathematics course identified per CCSSM standard. Because participants were able to select one or more mathematics course per standard, totals per CCSSM standard could be more than 23.

PSMTs identified Calculus most frequently as the course where relevant content was taught with regard to teaching secondary mathematics. Abstract Algebra and Linear Algebra were the second and third most identified courses. In particular, the selection of Abstract Algebra was interesting, as in anecdotal experiences, it is not uncommon to hear about a PSMT query the professor, "When will I ever use Abstract Algebra in teaching secondary mathematics?" Yet PSMTs identified receiving instruction related to their future when enrolled in Abstract Algebra.

Although Calculus, Abstract Algebra, and Linear Algebra were the top three courses identified by PSMTs, the choice of "None of the Listed Courses" was most frequent overall; "None" was the top choice in 14 of the 22 standards. For 11 of the 19 standards pertaining only to Grades 7 and 8, "None" was identified most often. In other words, for 58% of the early-secondary CCSSM standards, 23 PSMTs did not identify a mathematics course in which they discussed relevant

mathematics content pertaining to teaching and learning mathematics in the CCSSM.

Table 4.

Common Core State Standards Coverage by College Mathematics Courses

Grade	Abbreviated Course Names														
Strd	Clc	DM	LA	AA	Gm	PS	Lg	NT	Eqn	Hst	NA	RA	No	O	Tl
7															
RP	7	3	7	5	3	6	1	1	0	5	1	0	8	3	39
NS	3	4	5	6	1	3	0	1	1	1	1	0	10	4	26
EE1	7	6	8	7	1	2	1	1	0	3	1	1	10	3	38
EE2	8	8	6	5	2	8	0	0	0	2	1	0	7	4	40
G1	6	1	3	3	8	0	0	0	0	3	0	1	8	3	25
G2	10	1	1	2	6	2	0	0	0	4	0	0	6	3	26
SP1	0	0	1	1	1	10	0	0	0	0	0	0	9	2	13
SP2	1	2	1	2	0	7	0	0	0	0	0	0	10	2	13
SP3	1	2	0	0	0	10	1	0	0	1	0	0	9	2	15
8															
NS	2	3	3	4	1	0	0	1	0	0	1	1	14	1	16
EE1	7	3	5	6	1	2	1	2	0	2	1	1	11	3	31
EE2	3	1	5	2	2	2	0	0	1	1	0	0	13	1	17
EE3	4	1	11	5	0	0	0	0	1	0	0	0	9	2	22
F1	11	3	5	6	0	2	0	0	0	1	0	1	9	4	29
F2	9	2	2	5	0	1	0	0	1	0	0	0	11	3	20
G1	0	1	1	2	5	0	0	0	0	0	0	0	12	3	9
G2	4	1	1	1	3	0	0	0	0	4	0	0	10	2	14
G3	9	1	0	2	5	0	0	0	0	0	0	0	9	2	17
SP	0	2	0	1	0	4	0	0	0	0	0	0	14	2	7
9															
NRN1	3	2	0	4	0	0	0	1	0	1	0	0	13	2	11
NRN2	3	2	2	7	0	0	1	1	0	0	0	1	11	2	17
NQ	5	4	2	2	1	3	1	0	0	0	0	0	11	3	18
Totals	11	8	4	13	1	3	2	2	0	1	0	1	35	7	

[1]Note: The first column is organized first by grade, and then by standard abbreviation ("Strd").

[2]Note: The course abbreviations are as follows – Clc = Calculus; DM = Discrete Mathematics; LA = Linear Algebra; AA = Abstract Algebra; Gm = Geometry; PS = Probability or Statistics; Lg = Logic; NT = Number Theory; Eqn = Theory of Equations; Hst = History of Mathematics; NA = Numerical Analysis; RA = Real or Complex Analysis; No = None of the Listed Courses; O = Other; and Tl = Total Number of Courses Identified per Standard (excludes "No" and "O" columns).

In examining the data in Table 4, it is important to remember that each number indicates there were "*x*" instances in which respondents linked a course to a standard. For example, in the shaded cell, there were 5 instances where PSMTs identified teaching and learning related to *analyzing proportional relationships and use them to solve real-world and mathematical problems* (e.g., 7.RP) discussed or "covered" in their Abstract Algebra experience. Furthermore, with respect to standard 7.RP, one should interpret that there was a total 15 people (23 respondents minus 8 "No" responses) that identified 39 instances in which they recognized coursework connected to this

standard.

Discussion

The data reported in this paper are important for several reasons. First, the data provide one measure to examine if PSMTs receive instruction in mathematics content courses related to content they will teach. Second, the data indicate that the PSMTs responding to our survey generally do not recall being taught theoretical connections to K-12 mathematics in college mathematics courses. This finding contrasts with the goals given in the MET recommendations. Third, the data suggest that the exposure PSMTs receive related to secondary mathematics content connections should not be reserved for special or capstone courses. An earlier focus on secondary mathematics should be an integral part of each course. If the content in mathematics courses focused more purposely on conceptual connections to PSMTs' future career, methods courses could more deeply build on such deeply constructed understandings. Unfortunately, in a methods course, taken late in a PSMT's preparation, there is generally not enough time to accommodate such meaningful connections. Finally, because preservice teachers are not currently receiving instruction related to content they will be teaching in mathematics content courses, these data provide support for those programs for preparing secondary mathematics teachers that have or are considering pedagogical content courses for secondary teachers.

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NEGOTIATING KNOWING AND TEACHING: PRE-SERVICE TEACHERS' BELIEFS AND CONCEPTIONS ABOUT MATHEMATICS AND MATHEMATICS TEACHING

Vivian R. Moody
Western Kentucky University
vivian.moody@wku.edu

Described in this article are the methods and results of a longitudinal study that examined the mathematics teaching efficacy of middle-school pre-service teachers enrolled in the three-course sequence, commonly known as Mathematics for Elementary Teachers. Over the course of three semesters, this study examined pre-service teachers' beliefs about mathematics and their capabilities for knowing and teaching mathematics. The author sought to understand how pre-service teachers view themselves as mathematics students and how this viewpoint influences their conceptions about teaching mathematics. The results of this study show that pre-service teachers with high efficacy beliefs held on to those beliefs, whereas, pre-service teachers with low efficacy beliefs shifted their beliefs toward increased confidence in their ability to effectively teach mathematics.

As mathematics educators continually look for ways to improve the mathematical experiences of pre-service teachers, it is important to examine these prospective teachers' beliefs about mathematics and mathematics teaching. Research suggests that pre-service teachers' efficacy beliefs about mathematics teaching hinge on their prior experiences as mathematics students, which usually reflect traditional roles of teachers as *dispensers* of knowledge and students as *receivers* of knowledge. Moreover, these conceptions of mathematics and mathematics teaching tend to be manifested in practices of teaching mathematics (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992). By contrast, current mathematics education reform and mathematics teacher preparation programs emphasize mathematics instruction where students are active participants in constructing their own knowledge of mathematics, and teachers *facilitate* the construction of this knowledge. Hence, pre-service teachers find themselves constantly negotiating knowledge and teaching with regards to their beliefs about mathematics and teaching mathematics (Borko et al, 1992).

Theoretical Underpinnings

Teaching Efficacy Beliefs

Past and current research on teachers' beliefs and self-efficacy document the profound role of pre-service teachers' beliefs on mathematics teaching and learning. Bandura (1977) defined self-efficacy as individuals' viewpoints, perceptions, or judgments of their capabilities to succeed. Bandura's theoretical perspective on self-efficacy emphasizes two constructs: *personal teaching efficacy* and *teaching outcome expectancy*. The personal teaching efficacy can be defined as a

teacher's belief in their own knowledge, skills, and abilities to be an effective teacher. The teaching outcome expectancy can be defined as a teacher's belief in the notion that student learning hinges on effective teaching, and moreover, effective teaching can illicit student achievement and success regardless of external factors affecting the student. One caveat to consider is that pre-service teachers' efficacy beliefs are developed over time, and once these beliefs are developed, they are very resistant to change. According to Bandura (1986), "People regulate their level and distribution of effort in accordance with the effects they expect their actions to have. As a result, their behavior is better predicted from their beliefs than from the actual consequences of their actions."

Similarly, Green's (1975) theoretical construct of *beliefs* emphasizes that prior experiences and how beliefs are formed and held impact individuals' propensity to change their beliefs. For example, Green posited that if an individual holds core beliefs—beliefs that are central to an individual's identity—then it is very difficult for that individual to change his or her beliefs. Green's theory is "useful" for cultivating mathematics teacher preparation programs that influence pre-service teachers' beliefs about mathematics and teaching mathematics. For purposes of this study, the author extended Bandura's and Green's theoretical perspectives to considering how beliefs and self-efficacy are intertwined and as such play a significant role in pre-service teachers' developing content knowledge and teaching of mathematics.

Mathematics Knowledge for Teaching

Research suggests that many *practicing* teachers do not possess the breadth and depth of knowledge needed to teach mathematics effectively (Adams, 1998; Borko et al, 1992; Graeber, Tirosh, & Glover, 1989; Hill & Ball, 2004; Tirosh & Graeber, 1989; Wilson, 1994; Sowder, Phillip, Armstrong, & Schappelle, 1998). Overwhelmingly, this is the case for many pre-service teachers who first enter the classroom as practicing teachers without a deeper conceptual understanding of mathematical concepts. This lack of conceptual understanding may impact these teachers' ability to respond to students' questions and their ability to extend mathematics lessons beyond basic skills (Wilson, Floden, & Ferrini-Mundy, 2001). In addition, studies have shown that there exists a connection between teachers' knowledge of mathematics and student achievement (Hill, Rowan, & Ball, 2005). Hence, many mathematics education researchers have concluded that this connection is profoundly related to mathematics teachers' preparation programs (Hill et al, 2005; Rowan, Chiang, & Miller, 1997; Monk, 1994).

Although teachers' knowledge of mathematics is vitally important to their effective teaching of mathematics, research (Battisa, 1994; Borko & Putnam, 1995) suggests that teachers' conceptual knowledge of mathematics may be colored by their beliefs about mathematics teaching and learning. Further, some studies (Battista, 1994; De Mesquita & Drake, 1994) have found that there is a direct relationship between teachers' instructional practices in the classroom and their self-efficacy beliefs about mathematics. Hence, preparing teachers in areas of content and pedagogy is not sufficient to impact their teaching effectiveness in mathematics. Research indicates that pre-service teachers must acquire a deeper and richer knowledge of mathematics content and pedagogical content knowledge, and pre-service teachers must come to hold beliefs in these entities consistent with reform efforts in mathematics education (Battista, 1994; Borko & Putnam, 1995).

Methodology

In the Department of Mathematics, where the author conducted the present study, pre-service elementary and middle-school teachers are required to take a three-course sequence of what is commonly known as *Mathematics for Elementary Teachers*. The first course in the sequence focuses on number systems and number sense. The second course in the sequence focuses on geometry, and the third course in the sequence focuses on data analysis, probability, and rational numbers. In an effort to assess and evaluate the effectiveness of the three-course mathematics sequence, the author sought to determine whether pre-service teachers' perceptions of their own understanding of mathematics changed or remained constant toward the end of the mathematics sequence. Additionally, the author attempted to establish whether the pre-service teachers viewed their own understanding of mathematics as sufficient for effectively teaching mathematics and if this view changed or remained constant toward the end of the mathematics sequence.

Participants and Procedure

To help ensure the integrity of course content, all pre-service teachers enrolled in the three-course sequence used the same textbook and were engaged in classroom activities related to a particular set of topics. Also, regardless of the instructor of record for each course, the intent was to emphasize a social-constructivist classroom environment. As such, instructors of record shared mathematics learning activities and used questioning to emphasize small- and whole-group discussions and to facilitate discourse about particular mathematics topics.

Data were collected from over 200 elementary and middle-school pre-service teachers. At the beginning of the first course in the three-course sequence, pre-service teachers completed the

Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) and then again at the end of the third course in the three-course sequence. The MTEBI consists of 21 items and is a Likert-scale instrument that has five response categories: strongly agree, agree, uncertain, disagree, and strongly disagree. Thirteen of these items are classified as the Personal Mathematics Teaching Efficacy (PMTE) subscale, and eight are classified as the Mathematics Teaching Outcome Expectancy (MTOE) subscale (Enochs, Smith, & Huinker, 2000). The PMTE subscale addresses pre-service teachers' beliefs about their capabilities—specifically, their own knowledge and skills—to become effective mathematics teachers. The MTOE subscale addresses those pre-service teachers' beliefs about effective teaching honing students' mathematics achievement regardless of external factors that may influence student learning.

Since less research focuses on middle-school pre-service teachers' beliefs, the author randomly teased out, from the larger pool of participants, 10 middle-school pre-service teachers to study more in detail. The data and results shared in this article reflect the research conducted with this subset of pre-service teachers. Moreover, since the author specifically sought to understand the pre-service teachers' perceptions of their knowledge of mathematics and the consequent impact on their efficacy beliefs about teaching mathematics, the data and results presented in this article are drawn only from the portion of the PMTE subscale that focuses on knowledge for teaching. The demographics of the 10 pre-service teachers are as follows: all 10 are middle-school mathematics majors; seven are Caucasian females; one is a Bosnian female; one is an Asian female; and the final participant is a Caucasian male.

Data Analysis

The author used an inductive, analytical approach in conducting this study and in analyzing the data (Patton, 1990). Content analysis, in which the researcher looks for particular themes and patterns that emerge from the data, was one type of strategy the author employed to analyze the data. The author's intent during data collection and analysis was to focus on theory development and theory confirmation. Specifically, the author sought to determine the essence of pre-service teachers' beliefs from the perspective that the *synergy* between self-efficacy and beliefs impact how middle-school pre-service teachers view their ability to conceptually learn mathematics and teach it effectively.

Findings

The results presented below address the Personal Mathematics Teaching Efficacy (PMTE) of the *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI). Recall, this subscale of items focuses on pre-service teachers' beliefs about their capabilities to teach mathematics effectively and their beliefs about their knowing mathematics well enough to teach it effectively. The PMTE was scored as follows: strongly agree = 1; agree = 2; undecided = 3; disagree = 4; strongly disagree = 5. Therefore, lower mean scores yield higher pre-service teachers' mathematics teaching efficacy beliefs. Table 1 provides the middle-school pre-service teachers' mean scores in regards to their beliefs about teaching mathematics concepts effectively. Also provided are raw scores for each of the 10 pre-service teachers. Notice that pre-service teachers who strongly believed that their ability was sufficient to teach mathematics effectively tended to hold on to those beliefs throughout the three-course sequence. However, pre-service teachers who placed little confidence in their ability to teach mathematics effectively shifted their beliefs to *increased* confidence in their ability.

Table 1											
PMTE Item	**1st Course Mean Score**					**3rd Course Mean Score**					
I know how to teach mathematics concepts effectively.	Mean Score = 2.7					Mean Score = 2.1					
Pre-service Teachers	A	B	C	D	E	F	G	H	I	J
Raw Scores During (1st) Course	2	4	2	3	2	3	3	3	3	2
Raw Scores During Third (3rd) Course	1	2	2	3	2	3	2	2	2	2

Similarly, as shown in Table 2, the middle-school pre-service teachers who were confident in their own understanding of mathematics to teach it effectively, tended to hold on to those beliefs throughout the three-course mathematics sequence. By contrast, those pre-service teachers who were not confident about their own understanding of mathematics at the start of the three-course sequence shifted their beliefs toward the end of the three-course sequence and then tended to believe that they understood mathematics well enough to teach it effectively.

Table 2										
PMTE Item	**1st Course Mean Score**					**3rd Course Mean Score**				
I understand mathematics concepts well enough to be effective teaching elementary mathematics.	Mean Score = 1.9					Mean Score = 1.6				
Pre-service Teachers	A	B	C	D	E	F	G	H	I	J
Raw Scores During (1st) Course	1	2	1	3	1	2	3	2	2	2
Raw Scores During Third (3rd) Course	1	2	1	3	1	2	2	1	2	1

Shown in Table 3 below, findings with regards to pre-service teachers' beliefs about their own skills being necessary and sufficient to teach mathematics are presented. Note that the PMTE item is *negatively* stated. Therefore, for this particular item, higher pre-service teachers' raw efficacy scores imply higher teaching efficacy beliefs. Again, as shown in Table 3, the majority of pre-service teachers with *high* efficacy beliefs held onto those beliefs, and pre-service teachers with *low* efficacy beliefs tended to shift those beliefs toward increased confidence levels. However, two pre-service teachers' beliefs shifted from high efficacy beliefs to lower efficacy beliefs. At the beginning of the three-course mathematics sequence, pre-service teachers B and F did not *wonder* if they had the necessary skills to teach mathematics. However, as shown in bold print in Table 3, at the end of the three-course sequence, these pre-service teachers did *wonder* if they had the necessary skills to teach mathematics. Recall, these two pre-service teachers exhibited high teaching efficacy beliefs either at the beginning of the three-course mathematics sequence and held on to those beliefs, or their beliefs shifted to higher teaching efficacy beliefs toward the end the mathematics sequence (see Tables 1 and 2 above).

<table>
<tr><th colspan="11">Table 3</th></tr>
</table>

PMTE Item	1st Course Mean Score					3rd Course Mean Score				
I wonder if I will have the necessary skills to teach mathematics.	Mean Score = 3.3					Mean Score = 3.6				
Pre-service Teachers	A	**B**	C	D	E	**F**	G	H	I	J
Raw Scores During (1st) Course	5	**4**	5	1	2	**4**	4	2	2	4
Raw Scores During Third (3rd) Course	4	**2**	4	4	4	**2**	4	4	4	4

What is puzzling is how these middle-school pre-service teachers could hold beliefs that reflect their understanding of mathematics well enough to teach it, but could simultaneously hold beliefs that point toward their not having the skills necessary to teach mathematics. This conundrum leads to important questions. How do pre-service teachers attach meaning to what is necessary and sufficient to teach mathematics? Is it possible for pre-service teachers to believe that they know mathematics well enough to teach but at the same time believe that they lack the skills necessary to teach mathematics? Did the pre-service teachers in this study misinterpret this item on the PMTE subscale, and finally, how do pre-service teachers perceive their capabilities to attain these *necessary skills* to teach mathematics? Clearly, subsequent inquiry is necessary (and perhaps sufficient) to answer these questions!

Conclusion and Further Challenges

This research was designed to understand pre-service teachers' self-efficacy beliefs with regards to their knowing mathematics and knowing mathematics for teaching. Having conducted this study and other similar studies, it is the author's contention that middle-school pre-service teachers may enter their teacher preparation programs with a set of centrally held beliefs about teaching and learning mathematics that may differ significantly from elementary or secondary pre-service teachers. It has been well established in the literature that teachers' beliefs form over long periods of time, and changing teachers' beliefs may require even longer periods of time. However, less research has focused on middle-school pre-service teachers' beliefs. Thus, it is important to understand how and why, during their teacher preparation programs, some pre-service teachers hold on to their beliefs about mathematics teaching and learning, while others change their beliefs. The author purports that further inquiry, similar to the one presented in this article, needs to be

cultivated to harness a missing link to an increasing body of research on teachers' beliefs. Understanding middle-school pre-service teachers' knowledge and beliefs has the propensity to improve these prospective teachers' mathematical experiences in teacher preparation programs, which may ultimately impact the mathematics learning of their future students.

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ELEMENTARY TEACHERS' MATHEMATICAL CONTENT KNOWLEDGE, EFFICACY, AND PROBLEM SOLVING ABILITIES IN ALTERNATIVE CERTIFICATION

Brian R. Evans
Pace University, New York
bevans@pace.edu

The purpose of this study was to understand teachers' mathematical content knowledge, efficacy, problem solving abilities, and teacher beliefs in an elementary education mathematics methods course for special education teachers in alternative certification programs. Findings revealed a significant increase in mathematical content knowledge and teacher efficacy. Additionally, teachers were found to have high efficacy at the end of the semester and strong problem solving abilities. Teachers generally found helping students with disabilities learn mathematics was the biggest issue in their teaching, and the use of technology and manipulatives were the most important topics addressed in their learning.

The purpose of this study is to provide an understanding of teachers' mathematical content knowledge, efficacy beliefs, level of problem solving abilities, and teacher beliefs in an elementary education inquiry-based mathematics methods course for special education teachers in the New York City Teaching Fellows (NYCTF) and Teach for America (TFA) alternative certification programs. Understanding teacher knowledge is important because it is directly related to student achievement (Hill, Rowan, & Ball, 2005). Efficacy is a teacher's belief in his or her ability to teach effectively and positively affect student learning outcomes (Bandura, 1986; Enochs, Smith, & Huinker, 2000), and is an important component for successful teaching. Additionally, there has been an ongoing effort to help teachers teach mathematics using problem solving, and it is recommended mathematics should be taught through a problem solving perspective (NCTM, 2000; Schoenfeld, 1985).

Problem solving continues to be of high importance in mathematics education (NCTM, 2000; Posamentier & Krulik, 2008; Posamentier, Smith, & Stepelman, 2006). It is one of the five National Council of Teachers of Mathematics (NCTM) standards (NCTM, 2000), and is critically important in how students best learn mathematics (Posamentier et al., 2006). The National Council of Supervisors of Mathematics (NCSM) has considered problem solving to be the principal reason for studying mathematics (NCSM, 1978).

In order to understand what problem solving is, first it must be understood the definition of a mathematical "problem" must be understood. Charles and Lester (1982) defined a mathematical problem as task in which (a) The person confronting it wants or needs to find a solution; (b) The

person has no readily available procedure for finding the solution; and (c) The person must make an attempt to find a solution. According to Krulik and Rudnick (1989), problem solving is a process in which an individual using previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. Polya (1945), in his seminal work *How to Solve It*, outlined a general problem solving strategy that consisted of (a) Understanding the problem; (b) Making a plan; (c) Carrying out the plan; and (d) Looking back.

Understanding the level of elementary school mathematics teachers' problem solving abilities is critical in supporting them to teach their students from a problem solving perspective. For example, teachers are critically important in developing abstract thinking in students in the problem solving process. However, Cai (2000) has shown that sixth grade U.S. students rarely used abstract strategies in their problem solving. Strong problem solving abilities among teachers are needed if teachers are to teach mathematics well because content knowledge by itself, while being necessary, is not sufficient for good teaching (Ball, Hill, & Bass, 2005; Ma, 1999). The NCTM (2000) proposed, "Problem solving is not only a goal of learning mathematics but also a major means of doing so" (p. 52). If there is interest in good student problem solving, teachers need to be more than proficient in their own problem solving abilities.

The NYCTF program is an alternative certification program developed in 2000 in conjunction with The New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007). The program goal was to recruit professionals from other fields to supply the large teacher shortages in New York City's public schools with quality teachers. Prior to September 2003, New York State allowed for teachers to obtain temporary teaching licenses to help fill the teacher shortage. Teaching Fellows generally are recruited to teach in high needs schools throughout the city (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2006).

TFA is a non-profit organization formed in 1990 with the intention of sending college graduates to low-income schools to make a difference for the underserved students. Its founder, Wendy Kopp, was herself a new graduate of Princeton University looking to do something more with her life after graduation (Kopp, 2003). She considered many recent college graduates at America's top universities would consider teaching low-income students if given the opportunity. The idea was there should be a teachers' corps that would allow new graduates at top universities with an interest in teaching to quickly begin teaching students in underserved communities.

Theoretical Framework

The theoretical framework of this study is based upon the emphasis of the importance of content knowledge for teachers (Ball et al., 2005). Second, Bandura (1986) claimed teacher efficacy can be subdivided into a teacher's belief in his or her ability to teach well, and his or her belief in a student's capacity to learn well from the teacher. Finally, NCTM (2000) and Schoenfeld (1985) have emphasized problem solving as a way of teaching though an emphasis on problem solving as an important process standard.

Research Questions

1. What differences existed between teachers' mathematical content knowledge and concepts of efficacy before and after an elementary mathematics methods course?

2. What level of problem solving abilities did new teachers have in an elementary mathematics methods course?

Methodology

The methodology of this study involved quantitative methods. The sample in this study consisted of 24 new teachers in the NYCTF ($N = 9$) and TFA ($N = 15$) programs. Participants were enrolled in an inquiry-based elementary mathematics methods course for special education that involved both pedagogical and content instruction.

Teachers were given mathematics content examinations and efficacy questionnaires at the beginning and the end of the semester, and were given a problem solving examination at the end of the semester. The mathematics content examination consisted of 20 multiple choice items that measured knowledge of number sense, probability and statistics, measurement, geometry, and algebra, and was based on the PRAXIS mathematics examination (Educational Testing Service, 2005).

The efficacy instrument was adapted from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) developed by Enochs et al. (2000), and measured concepts of efficacy. The MTEBI is a 21-item five-point Likert scale instrument, and is grounded in the theoretical framework of Bandura's (1986) efficacy theory. The MTEBI contains two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) with 13 and 8 items, respectively. The PMTE specifically measured a teacher's self-concept of his or her ability to effectively teach mathematics. The MTOE specifically measured a teacher's belief in his or her ability to directly affect student learning outcomes.

The problem solving examination consisted of five problem solving situations as adapted from the literature given at the end of the course (Krulik & Rudnick, 1989; NCTM, 2000; Posamentier & Krulik, 2008). Each item was worth two points and possible scores ranged from zero to 10 points.

Results

The first part of the first research question was answered using the mathematical content knowledge test, and data were analyzed using paired samples t-test (see Table 1). The results of the paired samples t-test revealed a statistically significant difference between pretest scores and posttest scores for the mathematics content knowledge test, and there was a large effect size.

The second part of the first research question was answered using the MTEBI with two subscales: PMTE and MTOE, and data were analyzed using paired samples t-tests (see Table 1). The results of the paired samples t-test revealed a statistically significant difference between pretest scores and posttest scores for the PMTE, and the effect size was moderate. Further, the results of another paired samples t-test revealed no statistically significant difference between pretest scores and posttest scores for the MTOE.

Table 1.

Paired Samples t-Test Results for Content Test and Efficacy Test (PMTE and MTOE)

Assessment	Mean	SD	t-value	d-value
Content Test				
Pretest	75.00	16.151	-3.778**	0.85
Posttest	87.08	11.971		
PMTE				
Pretest	2.70	0.504	-2.575*	0.45
Posttest	2.91	0.478		
MTOE				
Pretest	2.69	0.691	-0.213	
Posttest	2.71	0.666		

N = 24, *df* = 23, two-tailed

** $p < 0.01$

* $p < 0.05$

Further, independent samples *t*-tests were used to determine if the participants had significantly better concepts of efficacy at the end of the semester as compared to a neutral value coded as "2" on the survey sheet (see Table 2). For the PMTE and MTOE the results of an independent samples *t*-test revealed a statistically significant difference between efficacy scores and neutral scores, and the effect size was very large.

Table 2.

Independent Samples t-Test Results for PMTE and MTOE Scores

Assessment	Mean	SD	*t*-value	*d*-value
PMTE Actual	2.91	0.478	-9.310**	2.69
Scores	2.00	0.000		
Neutral Scores				
MTOE Actual	2.71	0.666	-5.249**	1.50
Scores	2.00	0.000		
Neutral Scores				

N = 24, *df* = 23, two-tailed

Equal variances not assumed.

** *p* < 0.01

Descriptive statistics were used to answer research question two. At the end of the semester teachers had a mean score of 8.54 out of 10 possible points on the problem solving examination with a standard deviation of 1.615, which represents the teachers' level of problem solving abilities. This represents a fairly high level of problem solving abilities for the teachers.

Discussion

The results of this study are consistent with the finding of Palmer (2006) and Swars, Hart, Smith, Smith, & Tolar (2007), who found there was an increase in efficacy in terms of elementary preservice teachers' ability to teach well and their ability to positively affect student outcomes. However, Palmer (2006) and Swars et al. (2007) examined preservice elementary school teachers, while in this study teachers were inservice and enrolled in alternative certification programs. The results of this study were somewhat inconsistent with the results found by Hoy and Woolfolk (1990), who found a significant decline in beliefs to positively affect student learning outcomes during student teaching, which had been attributed to teachers' exposure to the realities of the classroom. However, in this present study teachers had an increase in this belief despite early

encounters with the realities of the classroom. Perhaps there is something different about alternative certification preparation that could explain this difference.

It was found teachers had relatively high problem solving abilities. This is in contrast to findings in the literature (Ball et al., 2005; Ma, 1999), and commonly held perceptions of teacher problem solving ability (Paulos, 1990). The translation of strong problem solving abilities into the classroom should be further investigated, and it is suggested that alternative certified elementary school teachers be evaluated for their implementation of their problem solving abilities into the classroom.

Ball et al. (2005) found teacher knowledge is an important variable in preventing the achievement gap from growing in high need urban schools. Additionally, efficacy impacts student learning (Soodak & Podell, 1997). Considering the large number of alternative certification program candidates who teach in high need schools, it is extremely important for the sake of these students that educational researchers understand teacher knowledge, concepts of efficacy, and problem solving abilities.

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USING THE MATHEMATICS SCAN TO NURTURE INSTRUCTIONAL QUALITY DEVELOPMENT: A PROFESSIONAL DEVELOPMENT OPPORTUNITY

Holly Henderson Pinter
University of Virginia
hollyhenderson@virginia.edu

Robert Q. Berry
University of Virginia
robertberry@virginia.edu

Temple A. Walkowiak
North Carolina State University
tawalkowiak@gmail.com

This work elaborates on common elements existing in high quality mathematics instruction and potential implications for betterment of classroom instruction through the use of the Mathematics Scan (M-Scan) as a tool for linking research to practice. An understanding of mathematics instructional quality requires an understanding of the quality of teacher-student interactions. Developmental theory and research suggests the interactions between the teacher and students in a classroom are critical to student development and learning, and hence, are central to the classroom quality (Morrison & Connor, 2002; Pianta et al., 2008). Examining the nature of these interactions explains the primary mechanism of their influence on learning.

High quality mathematics instruction is an overarching goal of practices supported by the National Council of Teachers of Mathematics (NCTM) guidelines. The *Principles and Standards for School Mathematics* (NCTM, 2000) are a critical starting point as teachers and researchers envision what "high-quality" instruction looks like. The Principles (Equity, Curriculum, Teaching, Learning, Assessment, and Technology) describe features of high-quality mathematics education. The Standards outline the mathematical content (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and the processes (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) in which students should engage and learn. Together, the Principles and Standards articulate a vision for ways in which mathematical content and pedagogy are blended and enacted in high-quality instruction. What is missing, however, is a way to examine the extent to which teachers implement instruction aligned with the vision of NCTM. To address this need, the Mathematics Scan (M-Scan), an observational measure of mathematics instructional quality, was developed. The ability to measure mathematics instructional quality stands out as a critical need in efforts to assess, and ultimately, improve instruction.

Building on the theory of the "mathematics teaching cycle," (NCTM, 2007) the M-Scan aims to operationalize how to capture *the implementation of mathematics instruction.* The focus of this study, the implementation of mathematics instruction is a crucial connecting piece between the other two components of the teaching cycle: the knowledge teachers bring to a lesson and their post-lesson analyses of their own instruction. To elaborate this concept we use a conceptual model describing the umbrella construct of Mathematics Instructional Quality, which is enacted through

mathematical teaching practices as having four domains: 1) the tasks that teachers select and the way in which these tasks are enacted in the classroom, 2) the discourse between teachers and students and among students about mathematics, 3) the representations used by teachers and students to represent and translate among mathematical ideas, and 4) the mathematical coherence of lessons including the demonstrated knowledge of teachers focusing on the extent to which the mathematical concepts are presented clearly and accurately and to which student misconceptions are addressed.

The Mathematics Scan

The M-Scan measures *Mathematics Instructional Quality* in a given lesson by assessing the aforementioned domains. To assess the mathematical tasks of a lesson, the M-Scan measures the dimensions of *cognitive depth*, *problem solving*, and *connections and applications*. To examine discourse, the M-Scan measures *explanation and justification* and *mathematical discourse community.* To assess representations, the M-Scan measures *multiple representations* and *students' use of mathematical tools.* To assess the mathematical coherence the M-Scan measures both *mathematical accuracy* and the *structure of the lesson.* The descriptive indicators for the nine dimensions were developed because of their link to NCTM's vision of high-quality mathematics instruction as well as the presence of their noted importance by researchers in the field of mathematics education (Borko et al., 2005; Stecher et al., 2006). These descriptive indicators specify in finer detail the elements we believe to suggest high quality mathematical teaching practices. For example, Mathematical Discourse Community (Berry et al. 2010) is comprised of three distinct features: Teacher use of discourse, student use of discourse, and questions. Within these indicators there are specific descriptors of high quality mathematics instruction. Take questions for example; this indicator assesses whether students are solicited for input regularly in classroom instruction as well as the questioning technique of the teacher. Does the teacher ask questions that elicit one-word answers or are more open-ended questions asked to focus on mathematical thinking? Each set of indicators supplies key words to suggest low, medium, or high presence of the given indicator. (While space limits further description of each dimension here, see Merritt, Rimm-Kaufman, Berry, Walkowiak, & Ottmar, 2010 for further detail.)

This work aims to address ways in which the M-Scan can be utilized as a framework for practicing teachers to link research to practice. The M-Scan has so far been utilized to quantify the quality of mathematical instruction in classrooms as defined by the four domains and nine

dimensions. We argue the potential to translate to practice as a diagnostic tool rather than a measurement tool. Instead of evaluating the quality of instruction the M-Scan will now serve as a framework for recognizing strengths and areas of need of enacted teaching practices. Often times the goals of professional development in mathematics focus on improving classroom instruction, but the goals may not provide opportunities for teachers to diagnose areas for improvement. Using the M-Scan as a framework for professional development provides practicing teachers the opportunity to target particular domain(s) (tasks, discourse, representations, or mathematical coherence) as areas for development in quality of instructional practice.

Professional Development

Much of the current research in teacher education supports the use of professional development as a vehicle for reform of educational practices (Cozza, 2010; Darling-Hammond 2005). According to Cozza (2010), good teachers must be proficient in these areas: "the content of the subjects they teach, research-based strategies to provide instruction, data to make instructional decisions, and differentiated instruction" (p. 228). All of these elements can be identified within the dimensions of M-Scan suggesting its promise, as a tool for addressing professional development needs. Sowder's (2007) work argues the lack of applicable and quality professional development opportunities stating: "Rarely do these in-services seem based on a curricular view of teachers' learning. Teachers are thought to need updating rather than opportunities for serious and sustained learning of curriculum, students, and teaching." (Sowder, 159).

In developing a professional development model to utilize along with the M-Scan we must address the crucial components of a successful mathematics professional development. According to Sztajn et al. (2011) the important aspects of effective professional development include teacher input and participation in decision-making, collaborative efforts, provision of strong models of instruction, and more long-term efforts as opposed to a singular workshop. Again, we find seamless correlations to using the M-Scan as a tool—the teacher will be participating in the identification and exploration of an area for development; math specialists, administrators, or other teachers can be called upon for support; and the M-Scan can be employed continuously over time to ensure consistency and to show growth in the quality of mathematics teaching practices offered by individual teachers.

Method

The present study uses data from a three-year longitudinal randomized control efficacy study of a socio-emotional learning intervention, the *Responsive Classroom® (RC)* Approach (Northeast Foundation for Children, 2010). Approximately 270 third, fourth, and fifth-grade teachers were videotaped on three or four occasions during mathematics instruction. These lessons were observed and coded using the M-Scan by trained coders (reliability on each dimension at or above 80%) using a likert scale. An analysis of lessons in our database of 750 lessons shows that "typical" lessons are often highly accurate; score "medium" on structure of the lesson, problem solving, cognitive depth, multiple representations, and discourse community; and score "low" on connections & applications, explanation & justification, and use of tools. (For more detailed descriptions of each dimension see the measure on the social development lab http://www.socialdevelopmentlab.org/). While the indicators differ for each dimension, the general guidelines for low, medium, or high represent rare presence, occasional presence, or often presence of quality instructional practices (ex. Low on problem solving equates to rare presence of grappling with mathematics and no presence of multiple strategies, versus a high score where students often grapple with mathematics and use multiple strategies to solve problems). The likert scores were then translated to categorical descriptors of low, medium, or high. A descriptive analysis of lessons in the database suggests that a "typical" lesson may have certain characteristics regarding dimension scores. For example, the domain of tasks the data suggests that most lessons score in the medium range for both structure and cognitive demand (70.3% and 63.9% respectively). However the dimensions of problem solving and connections and applications show an overwhelming majority of lessons to be in the low to medium range. Less than 10% of the lessons score high on problem solving, cognitive depth, or connections and applications. The second domain of representations show over 50% of lessons to fall in the medium category and approximately 5% of the lessons are scored high. Similarly 5.6% of the lessons score high on students' use of mathematical tools, while the majority of the lessons (61.4%) are in the low range. The discourse domain shows variation as well, with 61.7% of lessons scoring medium on mathematical discourse community, while 58% of lessons are scored low for explanation and justification. According to this data set only about 3% of the lessons score high in the dimension explanation and justification. Finally, the majority (62.3%) of lessons score high under demonstrated knowledge/mathematical accuracy. These statistics help to paint a picture of the distribution of these lessons and what a typical lesson might comprise.

Using this data as a profile, a lesson was selected having M-Scan scores of the typical lesson in 6 out of 9 dimensions. (It should be noted that no lesson was identified to represent the typical lesson criteria in 9 out of 9 dimensions.) With this data we can identify particular areas of focus for future instruction through the use of professional development. Using the specific indicators for each dimension within the domain of need, we can then identify specific strategies and behaviors that can support the growth and development within the domain of focus. This can be accomplished both for individual teachers based on data collected within their own classrooms as well as for small groups of teachers who have similar patterns in their areas of need.

Context of the Study

Ms. Green (pseudonym) is a young white female. She teaches a fourth grade class with twenty-one students. The presence of a special education assistant in the classroom suggests the presence of students identified for special education services, although specific information on this is unknown. In the observed lesson three adults were present in the room: Ms. Green, a special education teacher, and a special education assistant. According to Ms. Green's report to the videographer this lesson represents a typical day in her classroom and she feels as though her class objective for the day was met. The lesson lasts approximately 59 minutes and had the components of a warm-up, whole class instruction, and student exploration time. The content objective Ms. Green reports to the videographer is to address liquid measurement. The content objective for Ms. Green's lesson is to address liquid measurement. As a warm-up, students continue previous exploration of liquid measurement conversions using a pictorial representation known as the "gallon man". Students spend the first ten minutes of class on a worksheet of conversion problems such as "One gallon is equal to _ quarts." The class joins together in front of the Smart Board where Ms. Green presents a lesson entitled "The Big G." Ms. Green's lesson begins with both physical and pictorial representations of different liquid capacities. She models the amounts of each unit that are in a gallon while also giving different shape models for particular amounts. For example, while the capacity of two objects may be the same the dimensions of the model may be different, possibly suggesting a different capacity. After introducing each unit of measure, this lesson uses a story and a picture to represent the Kingdom of Gallon, a place that has four queens, each with two princes who each have two crowns to wear. The characters represent the number of quarts, pints, and cups in a gallon, respectively. The second half of the lesson is utilized for student exploration focusing on the creation of "The Big G" for each individual student. This is followed by a task requiring

students to match a unit of measure to an appropriate use (ex. Eye drop medicine is likely measured in milliliters rather than liters). Finally, students are asked to explain how the story and picture of the Kingdom of Gallon may help them understand volume through a written journal assignment to be turned in to Ms. Green. Students are assigned a short worksheet (6 problems) of homework problems addressing appropriate measures for specific contexts.

Discussion

Describing the strengths and areas for growth in the lesson, both numerically through M-Scan scores, and qualitatively through rich descriptions, provides information about Ms. Green's needs for professional development in each domain of the M-Scan. First, an examination of tasks outlines the cognitive demand of the activities chosen to convey information as well as the opportunities for students to grapple with and connect with the mathematical content. The tasks chosen did offer some procedures with connections in asking students to reason through appropriate units of measure in different contexts. Additionally, Ms. Green offered multiple concrete examples of each unit while providing students a context in which each unit may be utilized in their real lives. These connections and applications, often lacking in our larger data set, were present and clear in this lesson. The examples provided to students were relevant to their lives in addition to offering opportunity to connect to other mathematical ideas (such as the metric system). In this lesson, as with other lessons of Ms. Green's, discourse appeared to be an area that was rated low on the M-Scan. Both the dimensions of mathematical discourse community and explanation and justification scored low on not just this lesson but also the majority of the 7 lessons in our database for Ms. Green (4 and 6 out of 7 lessons respectively). This suggests that the discourse domain may be a relevant opportunity for professional development to aid in improving Ms. Green's instructional practices. Further analysis of her lesson indicates Ms. Green tended to direct the discussion of the class entirely, offering few to no opportunities for students to engage in student-to-student math talk. Additionally Ms. Green missed opportunities to encourage student participation through open-ended questions. Similarly, in the dimension of explanation and justification, we note that Ms. Green rarely asks students "how, what, or why" questions which would allow students to explain their thoughts, nor does she push students towards complex thinking. Third, there is a strong presence of representations in the lesson through the use of models and contexts, offering students multiple ways to think about and represent information needed for liquid measurement conversions.

Finally, Ms. Green demonstrates her mathematical knowledge by providing mathematically accurate information in a clear and concise manner, addressing student needs where applicable.

In thinking of this as a professional development model we would hope that someone in the role of mathematics specialist (or colleague or consultant) would now work with Ms. Green providing specific feedback and strategies based on M-Scan indicators. As previously mentioned, the dimension of Mathematical Discourse Community includes indicators of the teacher's use of discourse, sense of mathematics community through student talk, and questions. In this lesson Ms. Green tended to direct the discussion of the class entirely, offering few to no opportunities for students to engage in student-to-student math talk. Additionally Ms. Green missed opportunities to encourage student participation through open-ended questions. Similarly, in the dimension of explanation and justification, we note that Ms. Green rarely asks students "how, what, or why" questions which would allow students to explain their thoughts, nor does she push students towards complex thinking.

Through the analyses of each domain, we identify discourse as an area for professional development for Ms. Green. In the professional development, a school mathematics specialist, a district mathematics leader, or a university mathematics educator could work with Ms. Green by providing specific feedback and strategies based on M-Scan indicators in discourse. The dimension of Mathematical Discourse Community includes indicators of the teacher's use of discourse, sense of mathematics community through student talk, and questions. The dimension of Explanation & Justification includes indicators of both presence and depth of conceptual explanations and justifications for student thinking.

Initially, we suggest that the professional development provider follow up with Ms. Green in both self and guided reflections of the lesson. One way to meet this goal is to videotape the lesson (as we did in this case) and to ask Ms. Green to watch the lesson using the M-Scan framework to self-assess (which we did not have the opportunity to do in this study). Discourse between Ms. Green and the professional development provider would lead to agreement on specific indicators to focus future development. Together with Ms. Green, we would agree to focus on questioning as an area for development. Then, we would provide Ms. Green with specific questioning strategies (Chapin, O'Conner, & Anderson, 2003; Hufferd-Ackles, Fuson, & Sherin, 2004) and give her opportunities to rehearse these questions before implementing in her mathematics lessons. Additionally, both practitioner-based articles as well as research-based articles can be utilized to

provide Ms. Green with additional questioning strategies to implement into her daily instruction. Take for instance the article *Describing Levels and Components of a Math-Talk Learning Community* (Hufferd-Ackles et al., 2004), a research article that provides questioning strategies for effective instruction. This article highlights the use of probing questions, questioning focused on thinking rather than answers, and student-to-student interactions with teacher facilitation. Each of these components can be translated into practice through planning and implementation (Kazemi & Hubbard, 2008). The following course of development would focus on using M-Scan on intermittent lessons to provide continuous feedback over a period of time to support the ongoing development of Ms. Green's instructional practice.

Limitations

One concern of using the M-Scan framework as a professional development model is the scalability of the model—that is, the model requires that teachers work with consultants (colleagues, math coaches, etc) to examine the quality of the instruction, therefore may not be logistically possible large-scale. The strength of the model is the individualized support given to teachers as they identify needs for improving their practice. Regardless of large-scale implementation, this work can certainly be utilized at the minimum as a self-guided reflective practice for teachers to identify their own personal goals for improving their teaching.

We employ a case study approach to this piece, which could be viewed as a limitation in the realm of feasibility of educational research. Additionally, while not completely implemented with this participant, this model seems to be an appropriate starting place to utilize M-Scan in a professional development framework. Sowder states, "Case studies have become a useful way to understand and evaluate the effectiveness of professional development or intervention into the teacher preparation process." (p. 160).

Conclusion

We believe that the use of research tools in direct use with practicing teachers has great potential for influencing the development of instructional practices. Using the M-Scan as a measure provides differentiation as to the needs of individual teachers while remaining general enough to meet many needs. This suggests that the M-Scan can be feasibly utilized for larger groups of teachers; therefore, it may have a larger impact on the field of mathematics education as a means for improving the quality of mathematics instruction.

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RECASTING OF TRADITIONAL MATHEMATICS INSTRUCTION IMPROVES LEARNING

George Tintera
Texas A&M University-Corpus Christi
tintera@tamucc.edu

Ping-Jung Tintera
Texas A&M University-Corpus Christi
ptintera@tamucc.edu

"Unintended consequences of good teaching" was identified by Wagner and Parker (1993) as one of three possible impediments to the learning of algebra. Many problems related to linear equations can be recast to avoid reflexive and unnecessarily complicated solutions so as to lessen an impediment to the learning of linear equations. A switching replication study was used to demonstrate that this recasting will increase the accuracy of getting the correct answer, increasing score without increasing the time to study and decreases the time needed to learn and solve problems with this style of learning.

The study of algebra by freshman in college is an area in which the question of skill efficiency or conceptual understanding can be posed as alternative learning goals as done in Hiebert and Grouws (2007) in the context of learning number. They found that, in the case of teaching for skill efficiency, the teacher plays a central role in helping meet well-defined learning goals. The purpose of this study is to demonstrate that many problems related to linear equations can be recast with the result that students can improve accuracy in solving the problems as well as decrease the time needed to solve the linear equations with increased scores without increasing time in studying. This responds to the call of Star (2005) to generate a wider understanding of procedural knowledge. The result is that the locus of the mathematical content of the lesson is also a crucial component of having student meet specific learning goals.

Algebra textbooks (e.g., Sullivan & Sullivan, III, 2001) give example solutions to problems of finding the slope of a line represented by an equation. The equation is converted into slope-intercept form if it is not in it already, and then the coefficient of the independent variable is taken as the slope. The authors of this paper regularly challenge math educators to solve these types of problems and finds that the math educators, inevitably, take this approach in solving these problems. In this paper, the traditional approach is defined as the one where the slope of the equation is found from the slope-intercept form.

The novel method simply recognizes that the slope of a line whose equation is in general form, $Ax + By = C$, is the negative of the ratio of the linear coefficients in the equation ($m = -A/B$). Reversing that ratio and taking the opposite sign that gives the slope of the line perpendicular to the given one. This method is not new. This observation was made in Kindle (1950), but the complete solutions presented in that book followed the traditional method.

This paper compares student how students perform with two methods of solving problems involving parallel and perpendicular lines. One based on the traditional view of slope as linear coefficient in the slope-intercept form and the other as the negative ratio of the linear coefficients from the general form. In particular, this study addresses the question as to whether there is any difference on scores and time spent solving problems on a five-minute quiz given as both a pre-test and two post-tests. This study took place over a two week period showing that a conceptual change in instruction over a brief period could make a positive impact on student learning of mathematics.

Methodology

This section provides information on the sample, variables, the experimental design, the procedure, including instruments, and a description of the data analysis. Rather than focus on direct measures of understanding, we studied accuracy and time used to solve problems to allow for inference about the student achievement of particular learning outcomes. The experiment followed a Switching Replication Design (Shadish, Cook & Campbell, 2002) with the treatment being instruction in the novel method. The strength of this design is that the effect of the treatment is apparent in both groups, a control and experimental group. The score (0 to 5) on each of three levels of testing of subjects and the time needed to complete the items on each of the three levels of testing (from 0 to 5 minutes) served as dependent variables.

Sample

The sample consisted of n = 159 students in College Algebra in a regional university in the Southwestern United States. The students were taught by the same instructor in six sections with approximately 240 students enrolled. Those who did not complete a consent form or fully participate in the testing were excluded from the sample for this study.

Variables

Participants in this study were categorized into a control group or experimental group. The treatment in the study was instruction in the novel method, while the control group was instructed in the traditional method.

The factors for the study were the group (control vs. experimental), test (pre-, post- and 2nd-post) and participant id.

The two dependent variables were related to student performance on a five question, multiple-choice instrument limited to five minutes. Two items from the instrument are below in Figure 1. The first was simply the score (0 to 5) of the number of questions answered correctly. The second

was the time, in minutes, taken to complete the five questions. The time was self-reported by the students who read it from a large displayed timer in the classroom. These two measurements were taken three times, once on a pre-test, on a post-test and on a second post-test.

Find the linear equation from the given info.
3) Containing the point (1, -3) and parallel to -5x + y = 7   3) _____
   A) 5x+y= -8   B) -5x+ y= -8   C) -x+5y=-40   D) 5x+ y= 8

4) Perpendicular to the graph of 2x + y = -9; y-intercept (-1, 5)   4) _____
   A) -x-2y= 9   B) -2x+ y=7   C) -x+2y=5   D) -x+ 2y=11

Figure 1: Two typical items from the instrument

Design

Following the Switching Replication Design in this study, participants were assigned into the control and experimental groups by sections. The experimental group received the treatment of instruction in the novel method between the first two levels of testing, and the control group received the treatment between the second and third levels of testing.

The instructor assigned sections to either the control or treatment group by day of the week that classes met in the course schedule. This minimized the diffusion of the treatment through social effects. Completing the study over a short period of time (three tests over a two-week period) minimized the maturation effect.

Procedure

The study took place over three class meetings in a two week period and consent forms for the study were collected before the first day of the experiment.

On the first day of the experiment, all participants were given the pre-test. After the pre-test was collected material was presented by the same instructor to the two groups, traditional to the control and novel to the experimental. Students from both groups were assigned the same practice problems as homework after the first class meeting.

A post-test similar to the pre-test was administered at the beginning of the second class meeting. Instruction in the novel method was given to the students in the control group, and no further instruction was given to the students in the experimental group.

A second post-test, similar to the two previous tests, was given to all students at the beginning of the third class meeting.

Analysis

An ANOVA was tested using the aor routine in R, version 2.13.2. The factor variables were the group, the test and the participant id. The variables were the test score and test time.

Results

Table 1 shows the mean scores and mean time for each of the three levels of testing. It was expected that there would be an increasing trend for the scores and decreasing trend for the times for each group. These trends are apparent in the table.

Table 1.

Mean score (and time in minutes) used on each test.

	Pre-Test	Post 1	Post 2	Improvement, Pre to Post 2
Experimental	1.6	4.1	4.2	+2.6
Group	(4.7)	(3.3)	(3.3)	(-1.4)
Control	1.4	2.4	3.6	+2.2
Group	(4.7)	(4.8)	(4.0)	(-0.7)

Results for Score

A display of the means of the test scores over the three tests indicates that both groups had comparable scores on the pre-test. This is supported by the box plots for the two groups for that test. See Figure 2.
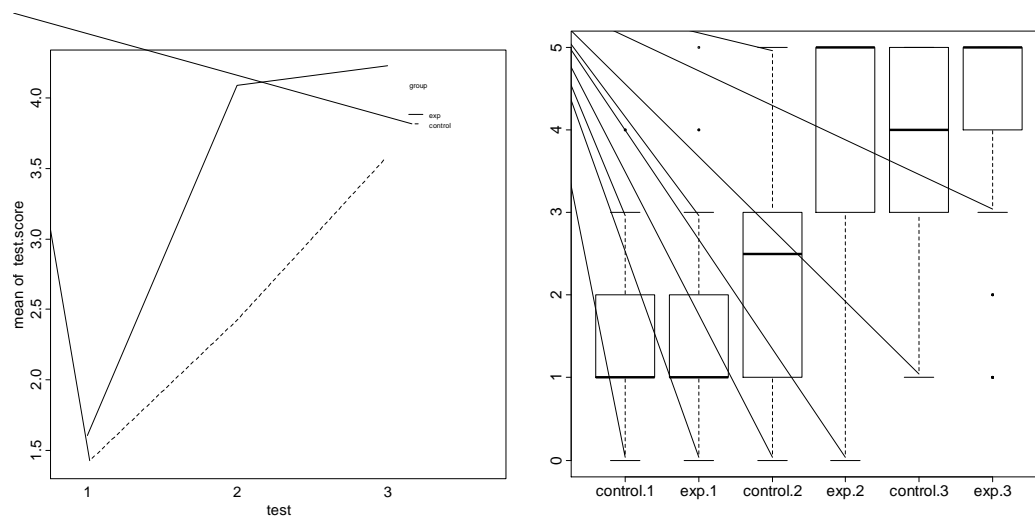
Figure 2

Between the first two levels of testing, the gain for the experimental group was greater than the for the control group. Comparing the box plots for the two groups for the first post-test (control 2 vs. exp 2) supports this claim. Also, comparing the box plots for the two groups supports the claim that each group had significant increases from the pre-test (test 1) to the first post-test (test 2). The gain for the experimental group was greater.

Between the 2nd and 3rd levels of testing, the control group increased its mean score, closing the gap with experimental group, and the experimental group showed a modest increase. The box plots support the claim that the control group test scores increased to level of the experimental group after it received its treatment, and that the increase of the control group was significant.

A repeated ANOVA test with factors group, test and id show that the effect of the instruction over time was significant ($p < 2.2e-16$), showing that both groups increased performance through the study. Also, the interaction of the group and id variables is significant ($p < 3.49e-10$) showing there was a difference in overall performance between the experimental and control groups. The test between groups showed that the apparent response of the experimental group shown by the higher trajectory in the test plot is significant ($p < 1.22e-9$).

Results for Time

The analysis above was repeated for the time used by the students to complete the problems. The first observation was that the mean times for each group on the pre-test were nearly identical. The results are shown in Figure 3.
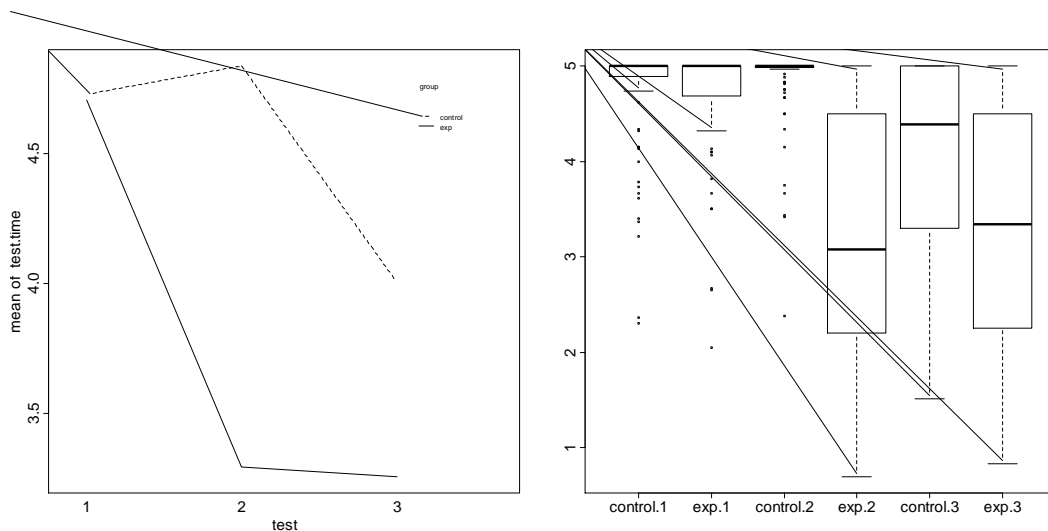
Figure 3

The experimental group decreased time needed to complete the instrument sharply after receiving the novel instruction. This is seen by looking at the decrease in the plot across tests as well as by comparing box plots for the experimental group from test 1 to test 2. The control group showed the opposite response to its first dose of instruction. There was a slight increase in time to close to the maximum time allowed (5 minutes).

The experimental group showed no further decrease in time from test 2 to test 3 while the control group showed a significant decrease in time.

The repeated ANOVA for the time variable was used to test with group, test and id as factors. The effect of the group on the time needed to complete the instruments was significant ($p < 2.2e\text{-}16$). Also, the interaction of the group and id variables is significant ($p < 1.06e\text{-}14$) showing there was a difference in overall performance from pre-test to post-test 2. The test between groups showed that the apparent response of the experimental group shown by the higher trajectory in the test plot is significant ($p < 4.86e\text{-}11$).

Discussion

The results support the idea that the instruction with the novel method had a substantial impact on both the achievement of students in terms of the scores on their instruments and the time needed to complete the instruments. This expands the notions expressed by Hiebert and Grouws (2007) that presentation of mathematical content has an effect on student learning.

The effect of instruction on time is worth discussing. The greatest effect on time to complete the instrument in each group was due to the instruction with the novel method. It might seem paradoxical that the effect of the instruction in the traditional method was to increase the time. But, this can be explained in that the traditional method itself is complicated, requiring greater attention of the student and time to use it correctly. Students in the control group were able to increase their scores, but only with increased time. While we would always like students to gain competence or understanding, it would be better that they do so in a shorter period of time. Given the methods of the study, we can only speculate that the time saved by students using the novel method was in not converting the given equation to slope-intercept form. Student facility with such equivalences would have to be fostered elsewhere.

The novel method is not in common use. The presence of the now traditional method in sources such as Kindle (1950) and many textbooks as well as the prevalence of functions in the current algebra curriculum may explain a preference for using slope-intercept form over the general form.

The results of this study show that the general form simplifies some calculations done with linear equations and, so, deserves a greater role in algebra courses.

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A THEORY-DRIVEN, CLINICAL PROCESS PARADIGM APPROACH TO DESCRIBING UNDERLYING BLOCKS TO TEACHING/LEARNING MATHEMATICS

George A. Pattison, III
[pka George A. Wyer]
Mathematics Education Researcher
bygapIII@comcast.net

Personal experience "clinicing" students of all ages this past decade has revealed some unusual sources of blocks to teaching-learning Mathematics. That most of the sources and blocks have little to do with Mathematics will probably not surprise anyone familiar with the history of the RCML [p.k.a. RCDPM]. Rather, it is the Clinical Process Paradigm employed, and patient search for underlying causes, that has produced results suggestive of further investigation. This paper will explore blocks associated with, but not limited to, 1) notational and visual perception issues, 2) missing Arithmetic and invented rules, 3) acquired Math phobia, and 4) reliance on step-by-step process.

When students achieve below expectation, what is done? "Teach-to-the-test"? Provide extra: credit, help (tutoring perhaps), time, and the like? Re-teach, in a manner that didn't work well the first time? Or acquiesce to student demand: "Just show me the step-by-step". No discussion, reasoning, decision making; no CHOICE!

Mathematical processing is essentially a *repeating* [looping], two-stage process of 1st making a CHOICE, and 2nd (only then) completing a step-by-step process. The CHOICE will invariably involve problem solving strategies, the basic *12 Rules of Mathematics*[1], and the use of hopefully appropriate – not invented – rules and strategies from the student's experience repertoire.

Not all blocks are directly observable, but still need to be accounted for in clinical intervention process and teacher training. Let's call these *Meta-blocks*.

What gets in the way of revealing what gets in the way of teaching/learning Mathematics?

I recently told our Mathematics Department that the 40 multiple choice questions on the Department final exam were not enough for content validity, and suggested something closer to 150. Everyone quickly agreed, but the Department settled on around 100 MC questions. Shortly thereafter I received a 30 question MC final. Oh well! So what's the problem?

Professors may now complete a tech-driven (perhaps bogus) item analysis, and then demonstrate "improvement" by teaching (dummying down) using step-by-step on only the five or six questions most missed. This limits or eliminates CHOICE; developmental Mathematics teaching need not occur! This is one of several reasons we need to retool our assessment practices in Mathematics teaching and learning. A second is overreliance on what amounts to mastery-related

assessment [often emanating from ridged use of the Percent Scale and the Bell Curve], rather than statistical sampling techniques. (I will save this for later.)

A third Meta-block involves class size. A local university administrator recently told me the class size for all their "developmental" Mathematics classes was capped at 30. I knew they filled these classes, and suggested a *developmental* class greater than 18-21[2] incongruous.

Thus, large class size, step-by-step-only instruction and inappropriate assessment are three major *Meta-blocks* that undermine sound Mathematics teaching, and unnecessarily overload clinical intervention efforts.

Finally, a conscientious clinician-teacher might administer a diagnostic content-test and remediate the specific questions missed. Time permitting, the instructor might try to "remediate" prerequisites as well as the target concept, perhaps even expanding on the original question scenario. This is often still not enough. Also, in my opinion, recent history suggests that over-reliance on technology and dummying down Mathematics instruction may exacerbate a decline in Mathematics teaching-learning effectiveness in the United States.

In truth, many dysfunctional and/or underachieving students simply don't make it through our fixer-filter. So, even if we assume we have provided the best available teaching-learning environment, with all the developmental bells and whistles a "guiding theory" on teaching-learning Mathematics might conjure, why do so many students still underachieve? What is missing? Are we focused too much on the Mathematics? What else is getting in the way?

A New Paradigm

My recent out-of-class clinic-style work, particularly with mathematically challenged university level students, is confirming the damage and providing a splendid opportunity for exposing underlying blocks to learning Mathematics, along with some unexpected sources of these blocks. What was needed was to patiently observe, listen, guide, and keep an open mind to non-Mathematics related blocks during one-on-one or one-on-two clinic sessions. The result was the following Paradigm for enabling a fresh approach to using *Clinical Intervention Process* in conjunction with teaching-learning Mathematics.

Theory-building has occupied a good deal of my professional career as an ancillary effort to understanding and improving the teaching-learning of Mathematics and concept development in particular. For example, this present exploration into uncovering and understanding new sources of

blocks to learning Mathematics is couched in a previously described comprehensive [unpublished] *Guiding Theory on the Interactive Process of Teaching-Learning Mathematics*[4].

Now I shall describe my theory-driven *Clinical Process Paradigm* to aid in describing underlying blocks to teaching-learning Mathematics, while also helping students achieve confidence in, control over, and success in their Mathematics processing.

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A *Mathematics Clinical Process Paradigm* (Theory)

I. DEFINITIONS [vocabulary]   II. AXIOMS [rules]

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I. DEFINITIONS

The *Clinical Intervention Process* model under development is open-ended in nature. That is, it is not bound by a priori lists or categories of blocks, nor limited to any particular rubric, research findings or methodology. I will refer to this open-ended mathematics clinical intervention process as **OMCIP**. Please note, this theory-driven process is meant to augment and not replace the extensive valuable research associated with clinical intervention process in Mathematics teaching and learning.

OMCIP is used with at most two students at a time, and seeks to overcome and possibly identify *underlying blocks* to learning Mathematics. While a viable tactic with any student, even an underachieving "genius ", the best first target source is often the bottom third of your class based on a pretest score, observed mathematical process, exam scores, work completion and/or unusual emotional distress. Three hours on OMCIP over two or three sittings is recommended. After three hours you may look for student gain with respect to confidence in, control over, and success in Mathematics processing, as evidenced by class participation and quality of work.

OMCIP cannot happen in a vacuum, and classroom discussion-observation is not enough. The instructor needs to monitor student process by 1) reviewing process on out-of-class assignments, 2) assessing student work using exams, collected class exercises, individual interviews, and the like, 3) observing exceptional or distracting behavior and 4) fostering sound lecture-discussion note-taking practice (dialogue) by requiring the maintaining of a *Portfolio Notebook*. The *Portfolio Notebook* enhances OMCIP session effectiveness and course success.

Intervention in applying OMCIP is straightforward. The Mathematics Clinician should listen, observe, and - buoyed by his or her teacher training and teaching experience - utilize a *guided discovery*, interview technique - much in the spirit of Kahlil Gibran - by leading the student to the

threshold of his or her own mind[3]. <u>If</u> identifiable, BLOCKS to Mathematics processing should be understood by the student and recorded.

Domains of intervention need to extend beyond the traditional Cognitive, Psychomotor, and Affective. There are three more domains I have identified over the years as relevant. In 1972 I added "Social Domain", and more recently, Environmental and Spiritual. Thus, **S.C.A.P.E.S**.

Finally, anxiety should be reduced when, for example, a student learns there are only *12 Foundation Rules of Mathematics* that govern most of the Mathematics they will encounter, or when a cliniced student starts to cleanly organize his or her work on paper, learns to correctly choose and apply proper rules, and is able to self-evaluate progress along a solution path.

II. AXIOMS [assumed rules]

The assumptions that support my *Mathematics Clinical Process Paradigm* follow. Examples of related observed blocks accompany each axiom.

Axiom #1 [Non-Mathematics Related Blocks]

Most blocks may have little or nothing to do with Mathematics. How often do we overlook "cultural differences" in students? If they can't do your algorithm, perhaps you should let them teach you theirs?

After observing scores of students in clinic settings, I suspected lines on paper could actually lead to errors. Students were restricted spatially, and limited visually. *Lines* constituted a fairly significant learning block. Consequently, I now require plain, unlined paper, for all phases of Mathematics instruction, with the left 2/3 of each page used for teacher directed content, and the right 1/3 of the page for student comments and related musings.

Lined paper is a carry-over from "English", and left-to-right processing. Mathematics is not about "left-to-right" processing. Mathematics involves vertical processing. "Equals added to equals the results are equal" requires vertical processing. Picking up and tracking the equation rightward and between the lines can foster error, be tedious, and lead to a proliferation of run-on sentences, even in textbooks. Not good! Here is a textbook example:

Example 1. Actual "horizontal ($\leftrightarrow$) sentence" from textbook:

$$A = \pi r^2 = (3.14)(5)^2 = (3.14)(25) = 78.5 \text{ ft.}^2$$

This is a run-on sentence, and is missing the "$\approx$" relation. It is a visual nightmare compared to the following (edited) vertical ($\updownarrow$) presentation:

$$A = \pi r^2$$
$$\approx 3.14(5)^2 \text{ ft.}^2 \quad \text{or} \quad \approx 3.14(25) \text{ ft.}^2$$
$$\text{So,} \quad A \quad \approx 78.5 \text{ ft.}^2$$

As we often see – and overlook - authors and textbooks take notational and process liberties which then become part of the Mathematics culture. Students mimic this practice, perpetuating underlying confusion leading to uncertainty and errors. For example:

Example 2. Consider how many handle positive and negative integers. Again, this is about symbol-conflagration and not Mathematics! Textbooks (and teachers) often present negative six and positive six as "–6" and " 6". The first notation says "subtract six". But subtraction is a *binary* operation. It should read "⁻6". The positive six notation omits the "⁺" asking the reader to assume that if the positive sign is missing, it means positive and not negative. Does the learner's "number sense" suggest it behaves the same as its natural number counterpart?

Here is a typical textbook rendering of a polynomial being evaluated for $x = {}^+4$ and $y = {}^-2$:
$$-2x^2 + 5xy - y^3 = -2 \cdot 4^2 + 5 \cdot 4(-2) - (-2)^3$$
$$= -2 \cdot 16 + 5(4)(-2) - (-8)$$
$$= -32 + (-40) - (-8)$$
$$= -72 - (-8)$$
$$= -64$$

The phrase became a sentence at the onset. The negative integer notation incorrectly implies subtraction. An "or" or "↔" should have been used instead of "=". Distracting parenthesis abound! Did the author mean there to be no negative integers? This doesn't need to be inherently confusing to be Mathematics. Remember, special font symbols "=" vs. "≈", "–" vs. "⁻" were readily available decades ago. What are we doing?

Example 3. So, who gets confused over negative integer (unary) vs. subtraction (binary) notation? Ask anyone to simplify (⁻6)(⁻6). Students can understand that "If a bad guy leaves the club, it is good for the club!", so all responses should be ⁺36.

Now ask a class to simplify -6^2. Surprise! No agreement. The *expected* answer is ⁻36, but the correct answer has to be ⁺36. If the "–" sign means integer, and not subtraction, then the "–" sign *belongs to* the "6". Thus, you are simplifying (⁻6)² which is (now) clearly ⁺36. Order of operations cannot justify this confusion as the first and only indicated operator is exponentiation (no subtraction). Do we not want students to think about this?

Example 4. Consider this textbook example.

$$\text{Let } x = 6. \text{ Then: } -x^2 = -6^2 = \mathbf{-6 \cdot 6} = -36 \quad \dots \text{ Fudge!}$$

Immediately, this is a run-on sentence, fudged, and based on faulty notational assumptions. The corrected simplification follows, clarified using parenthesis and proper integer signs.

$$\text{Let } x = 6. \text{ Then: } (-x)^2 = (^-6)^2$$
$$\text{or } (^-6)(^-6)$$
$$\text{or } {}^+36$$

Example 5. A final example on the matter of indiscriminately modifying or omitting certain symbols without regard for the developmental consequences. This involves the students understanding of the mixed numeral form, $\mathbf{5\frac{2}{3}}$. Pose the following question to the <u>class</u>:

$$\text{Is } \mathbf{5\frac{2}{3}} \text{ an indicated sum or product?}$$

$$\text{Do we insert a “}\mathbf{+}\text{” sign or a “}\mathbf{x}\text{” sign between the } \mathbf{5} \text{ and the } \frac{2}{3} \text{ ?}$$

I surprised four different classes with this question. The resulting vote was always close to a 50-50 split. Just what was their understanding of $\mathbf{5\frac{2}{3}}$? What is your understanding? Do we need an "inventiveness quotient"? Is the issue Mathematics-related or inappropriate use of notation?

Axiom #2 [Unlimited Blocks]

Blocks to learning Mathematics are unlimited, as they can emanate from any of the six aforementioned domains [S.C.A.P.E.S.]. They can also be unique to the student. This is the main reason for defining *Mathematics Clinical Intervention Process* as "OPEN". Here is an unusual example from an otherwise very reliable technology source.

Example 6. This is the only error (or disagreement) I have ever experienced with the marvelous visualizer-graphing utility that has done so much for the developmental treatment of Mathematics. Enter each of the following in your *TI84*, noticing you cannot use the "subtract" binary operator. The first response should be " $^+\mathbf{49}$ " for the reasons stated earlier. But ….

$$\text{First: } {}^-7^2 \ \rightarrow \ {}^-\mathbf{49} \qquad \dots \text{ oops!}$$
$$\text{Second: } (^-7)^2 \ \rightarrow \ {}^+49 \qquad \dots \text{ OK}$$
$$\text{Third: } {}^-(7^2) \ \rightarrow \ {}^-49 \qquad \dots \text{ OK}$$

While a technology-related block, in my opinion the latter pales in comparison to the problems being created by some contrived, quick-and-dirty, dummied down Mathematics software that gives the illusion of sound teaching/ learning Mathematics.

Axiom #3 [Self-Corrected Blocks]

Students may self-correct without the clinician necessarily identifying the source or block. Furthermore, in "leading the student to the threshold of his or her own mind", actual block sources could even remain unknown. Who has not asked a question of a teacher and self-corrected their issue, with their correction seemingly having no real bearing on the teacher's explanation? Perhaps that is why we can often get more creative work done during a sermon than when left alone! (Not relevant?)

Axiom #4 [Invented Blocks]

There are some Math rules students know; there are many they don't know; but there are also quite a few they invent that don't work! It is the later that seems to present the more interesting source for uncovering new blocks to learning Mathematics. Experience suggests hypothesizing that if you eliminate [using OMCIP] the erroneous rules, what remains could possibly constitute for many the desired viable Arithmetic-foundation sought! Of course, inventing rules is not the main problem. It is failure validate that hurts.

Example 7. Here is a common "invented-rule" example from a first year university student. As required, the student wrote this down to show process. It's wonderfully typical!

$$\text{Solve for the variable.} \quad \frac{5}{x+7} = \frac{3}{x+3}$$

$$\frac{5}{7x} = \frac{3}{3x} \qquad\qquad \text{Invented Rule: } x + 7 \leftrightarrow 7x \quad \text{… ouch!}$$

$$5(3x) = 3(7x) \qquad\qquad \text{Rule: (x)Prop(=) \underline{twice}, etc. OK}$$

$$-15x = -15x \qquad\qquad \text{Rule: (-)Prop(=) OK, but why? … process?}$$

$$x = 6 \qquad \text{Multiple process errors } \frac{15}{-15} \neq 1 \text{ ; } 21x\text{-}15x \neq 6 \text{ ; etc.}$$

NOTE: The student did *not* check by Substitution(=). $\quad \frac{5}{13} \neq \frac{3}{9}$

Perhaps one of the most popular and pervasive invented (and very bogus) rules is *cancelling across addition*. Combine this with cancelling like symbols - anywhere - and we have a winner!

Example 8. Simplify, if possible. $\qquad \dfrac{8x^2 + 32x + 12}{16x + 8} \quad \leftrightarrow \quad x^2 + 12 \quad \text{… very oops!}$

How'd he do that? What gets in the way is definitely not always necessarily related to Mathematics, and may in fact be invented, and perhaps even unique to that student.

Future Related Needs

In addition to describing and developing this *Mathematics Clinical Process Paradigm*, my research suggested the need for a) assessment reforms, b) curriculum realignment (textbooks and technology) and c) modified teacher training. To this end, I defined a *Master's Degree* program[5] promoting a certification in *Open Mathematics Clinical Intervention Process* [OMCIP]. These moves are essential for effective teaching-learning of Mathematics.

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Endnotes

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[1] The basic *12 Foundation Rules* that drive most of the Mathematics the student is likely to encounter are: the Field Postulates: closure, associativity, identity, inverses, commutativity, distributivity; Equivalence Relation Rules: reflexive, symmetric, transitive; and Equation Solving Rules: (+) Property(=), (x) Property (=), Substitution(=). Consistent reference to these *Rules* helps focus a student on what is essentially the "Arithmetic" developed in grades K-7.

[2] I conducted a Meta-analysis of class size research, and concluded the optimal developmental classroom should contain no more than 18-21 students. Unpublished. (Circa 1980).

[3] "On Teaching: … The teacher who is indeed wise does not bid you to enter the house of his wisdom but rather leads you to the threshold of your mind."

[4] The pursuit of this Guiding Theory-definition was originally inspired by a conversation with **Vincent J. Glennon**, University of Connecticut, after his *First Annual Dr. John W. Wilson Memorial Address at the Seventh Annual Conference of the Research Council for Diagnostic and Prescriptive Mathematics* (Vancouver, BC, April 13, 1980).

[5] I have outlined a *Master's Degree* program for **OMCIP**, and am seeking to implement same.

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AN OVERVIEW OF STEM EDUCATION PROJECTS IN THE UNITED STATES: CHARACTERISTICS AND CONCERNS

Alan Zollman
Northern Illinois University
zollman@math.niu.edu

This paper is an overview of Science Technology Engineering and Mathematics (STEM) Education projects in the United States in the past decade. What do successful STEM Education projects look like? What are the characteristics of successful afterschool programs for students? What are the attributes of successful projects that help teachers experience, plan, teach, reflect and transform the new content knowledge and pedagogical skills from the projects into their own teaching? From these projects, can we identify the areas of future needs for STEM Education professional development?

STEM Education, namely science, technology, engineering and mathematics education, is a current priority in the United States (Educate to Innovate, 2010). The reasons for being a current priority are twofold: first is the supply pipeline shortage – the necessity for more scientists, technicians, engineers and mathematicians – and second is the necessity for an educated population – more knowledgeable, innovative workforce. But these concerns have existed since World War II when the US feared the competitive challenges in science from Germany and Japan (Bush, 1945). To address these needs, the US has initiated recommendations and projects for what schools should do to solve these deficiencies (National Academies, 2005). This paper is an overview of STEM Education projects in the United States in the past decade to identify characteristics of successful projects and concerns for its future.

Background

We are now in the STEM Generation (Zollman, 2012). STEM Education has extended beyond the four core subjects to include such areas as agriculture, environment, economics, education and medicine (Zollman, 2011a). The US Government's intention for STEM Education is to resolve three needs: (a) societal needs for new technological and scientific advances, (b) economic needs for national security, and (c) personal needs to become a fulfilled, productive, knowledgeable citizen. The US Government has three major economic initiatives to address these concerns: through the National Science Foundation (NSF) grants; through the US Department of Education Mathematics-Science Partnership (DOE-MSP) grants; and through the Eisenhower Professional Development grants (now defunct). Analyzing a multitude of math-science and STEM projects, Zollman (2011b; 2012) identified seven distinct measures of these STEM projects: (a) student engagement, (b) student affective domain measures, (c) student STEM content knowledge, (d)

teacher STEM content knowledge, (e) teacher STEM pedagogy knowledge, (f) teacher affective domain measures, and (g) teacher practices in the classroom. *Student engagement measures* range from counting the number of STEM area graduates produced, to counting the number of students taking higher-level mathematics and science courses. *Student and teacher affective domain measures* include such things as motivation, self-esteem, self-confidence, attitudes, beliefs, and self-identity. *Student and teacher STEM content knowledge measures* are appraised by a variety of project-made, standardized, and state-mandated tests. *Teacher STEM pedagogy knowledge measures* refers to knowledge, skills and abilities in the teaching of such things as scientific inquiry, problem-solving mathematics strategies, problem-based learning, and reverse engineering. Finally, *teacher practice in the classroom measures* observe and evaluate the transfer of project objectives into the real world of schooling.

Methodology

The resources and publications of both the National Science Foundation and the US Department of Education were primary resources for this project analysis. Additional information was obtained though *MSPnet* created and facilitated by the Center for School Reform at Technical Education Resource Center (TERC), and identified published conference proceedings and journal articles that reporting on NSF and DOE projects. Follow-up interviews occurred at regional conferences of the Mathematics and Science Partnership Programs as well as at various universities and national conferences.

Findings Pertaining to Student Afterschool Connections

The term "afterschool" in this paper refers to before-school, after-school and summer learning options of students – opportunities for STEM enrichment experiences outside of the normal classroom instruction. The only central connections of these many, many enrichment opportunities are that these activities pertain, in some form, to either/or science, technology, engineering or mathematics topics. The activities vary from afterschool speakers to summer work internships; from once-a-month science club meetings to intensive summer-long boarding camps at universities. All have short-term goals of improving student affective-domain beliefs, attitudes and identity; academic knowledge and skills; and long-term goal of increasing the number of students studying STEM subjects and pursuing STEM careers.

A review of the evaluation reports by Afterschool Alliance (2011) found that "high-quality STEM" afterschool programs yielded specific benefits such as improved attitudes towards STEM

fields and careers, increased STEM knowledge and skills, and higher likelihood of pursuing a STEM career. The report concluded that afterschool programs may play a key role in engaging students from diverse communities into STEM fields and careers. Since most, if not all, of the students in afterschool programs are self-selected, finding improved attitudes and increased knowledge is not surprising. Thus it is difficult to perform a critical analysis of these projects to see if their results would transfer to situations without the proper funding, dedicated mentors and eager student participants. It may be more useful to analyze the in-school programs having teacher connections.

Findings Pertaining to Teacher Connections

The US Department of Education (Abt Associates Inc., 2010) in its *2008 Annual Report* investigated 626 Math-Science Partnership (MSP) projects between high-need school districts and institutes of higher education. This report found an increase in content knowledge (67% in math and 73% in science) involving 57,000+ teachers and a resulting 13% increase in mathematics and a 9% in science proficiency levels involving their 2.7 million students. The report credited these increases to the intensive (an average of 97 hours) and sustained content-rich professional development that integrates the content areas of mathematics and science with effective pedagogical knowledge. Further, the report pointed out that the MSP teachers received ongoing mentoring and coaching as they implemented their new knowledge and practices.

In a similar method, the National Science Foundation commissioned an analysis of 200+ STEM education reports and research articles written since 1995 (National Commission on Teaching and America's Future, 2011). NSF found that STEM teaching is more effective and student achievement increased when teachers develop strong professional learning communities in their schools. This report, and a one-year earlier report, *Team Up for 21st Century Teaching,* (National Commission on Teaching and America's Future, 2010) stated that teams of teachers were able to create a "culture of success" leading to student learning gains. These two NSF reports advocated for teams (Professional Learning Communities) of teachers to create schools that look like learning organizations with six principles: 1) shared values and goals; 2) collective responsibility; 3) authentic assessment; 4) self-directed reflection; 5) stable settings and 6) strong leadership support.

A third large-scale sample analyzed the Eisenhower Professional Development Programs (Garet, Porter, Desimone, Birman, & Yoon, 2001). Garet, et al., examined the correlations between professional development aspects of the various programs and teacher outcomes of knowledge,

skills and teaching practices. This meta-analysis found a slight positive effect of "reform-minded" professional development activities with an increase in teacher content knowledge and pedagogy skills, and further, this increase in teacher knowledge and skills was coupled with positive changes in instructional practices.

Conclusions

In these meta-studies cited, as in several papers prepared for the National Research Council *Workshop on Successful STEM Education in K-12 Schools* (Wilson, 2011; Schmidt, 2011), three common characteristics of successful STEM programs emerge:

- A strong focus on developing teacher knowledge of and ability to teach the subject matter;
- A solid relevancy to the teacher's classroom situation; and
- An intensive, sustained duration for the professional development.

These three characteristics fit with the consensus for "general" high quality professional development by the National Staff Development Council (2001) as well as specific guidelines of math-science professional organizations from the *National Science Education Standards* (National Research Council, 1996) and the National Council of Teachers of Mathematics *Professional Teaching Standards for Teaching Mathematics* (1991).

But in his study, Zollman (2011b; 2012) identified three areas of specific concern in STEM Education. He pointed out there is a need for the professional development to be organized, with consistency in the design and measures for success, and a long-term focus on building upon others' research rather than a continual re-invention of the projects. Second, teachers need easy access to research results that have been categorized by levels of evidence (e.g., an opinion paper versus a randomized trial) and a critique of the quality of the study. Third, in order to fully understand the effectiveness of various STEM professional development projects, projects need to include the quality of implementation/fidelity measures as well as growth model approaches to track teacher and student changes within specific environments.

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DO YOU SEE WHAT I SEE? AN EXPLORATION OF
SELF-PERCEPTION IN THE CLASSROOM

Eileen Durand Faulkenberry
Texas A&M University - Commerce
Eileen_Faulkenberry@tamu-commerce.edu

Thomas J. Faulkenberry
Texas A&M University - Commerce
Thomas_Faulkenberry@tamu-commerce.edu

Previous research has established a tenuous relationship between what teachers say they do and what teachers actually do in classrooms. In a 3-year longitudinal study, we followed a group of middle-school mathematics teachers in a professional development project. We investigated the relationship between these teachers' self-ratings of standards-based classroom practices and the ratings of a trained observer. We found that over time, the teachers shifted from overestimating the frequency of their standards-based classroom practices to underestimating their standards-based behaviors. The professional development project lessened the gap between what teachers say they do and what they actually do in the classroom.

While the current classroom environment is centered on assessment, there is typically less concentration on teacher self-assessment. Indeed, teacher self-assessment has been found to be somewhat lacking in reliability and validity (Ross, 2006; Burstein et al., 1995; Hook & Rosenshine, 1979). On the other hand, teacher self-assessment has been used positively as a method for promoting teacher efficacy (Ross & Bruce, 2007). Given the equivocal nature of claims regarding teacher self-assessment, especially in relation to assessments from outside observers, we set out to investigate the relationship between teacher self-assessment and actual classroom practices.

Generally speaking, there is a questionable relationship between the classroom practices that teachers report and their actual classroom behaviors. Frykholm (1996) collected 153 classroom observations of 41 preservice mathematics teachers. He found, quite paradoxically, that preservice teachers were readily able to give explanations for why they couldn't use standards-based classroom practices, while simultaneously reporting a high degree of symmetry between the standards and their own teaching. In fact, in a review of research comprising data from over 2300 teachers, Hook and Rosenshine (1979) asserted that "we cannot assume that these [self-] reports correspond to actual practice" (pp. 9-10).

 In a more recent study, Ross, McDougall, & Hogaboam-Gray (2003) developed a survey based on several dimensions of standards-based teaching. While they were able to demonstrate that several high scoring teachers were indeed implementing standards-based teaching in their classrooms, they also found that there was still quite a discrepancy between the teachers' perceptions of their standards-based teaching and their actual teaching behaviors. The authors concluded that this discrepancy was due to two main factors: (a) they did not understand what

standards-based teaching meant, or (b) they could not figure out how to implement their ideals into classroom practice. Given that professional development can ameliorate such concerns, it would be interesting to ask whether the same discrepancy between perceptions and actual behaviors exists even after extensive professional development.

The present study was designed to assess the effects of a long-term professional development program on teachers' perceptions of their abilities to teach mathematics using standards-based classroom practices. Critically, we chose to examine this through a comparison of self-reports of standards-based classroom behavior and the reports of a trained classroom observer.

As the previous literature in this area is sparse, predictions are difficult to formulate. However, we did expect to find that, compared to outside observation, teachers tend to overestimate the frequency of their own standards-based classroom behaviors. It is not clear whether this overestimation would persist throughout the duration of a prolonged professional development program.

Method

Participants

Five mathematics teachers from the Northeast Texas region participated in the study. The teachers were participants in a long-term professional development project directed by the principal author. Two of the participants were teachers in a suburban district. The three remaining participants taught in primarily rural districts. The grade levels represented ranged from Grade 4 to Grade 8. The mean years of teaching experience was 9 years (SD = 6.7 years, range 2-17 years) at the beginning of the study (2009).

Materials

Participants' standards-based classroom practices were assessed using a classroom observation protocol consisting of 23 items, each representing a statement of a teacher-action or a student-action (Papakonstantinou & Parr, 2004). Example items include "Teacher asks a variety of questions" and "Students are encouraged to explain the process used to reach a solution." Items are scored on a 5-point Likert scale indicating the degree to which a statement occurs during a lesson (1 = low frequency, 5 = high frequency).

Procedure

Over the course of three school years (2009, 2010, 2011), the participants' standards-based classroom practices were assessed in two different ways. First, participants were observed teaching

a lesson in their classrooms by a trained outside observer who completed the classroom observation protocol at the conclusion of the lesson. The same observer rated each participant during each year of the study. Second, each participant then completed the classroom observation protocol as part of a reflective prompt about the lesson they taught. This self-rating was completed within one day of the teaching of the lesson.

Results

For each participant, a mean of 23 classroom observation protocol items was recorded in each of 6 conditions defined by crossing the factors of Year (2009, 2010, 2011) and Rating Type (self-rated, observed). These means were subsequently analyzed using a 2 x 3 within-subjects analysis of variance (see Table 1). Comparisons between means were made using 95% confidence intervals, which were calculated using the method of Loftus and Masson (1994).

The analysis of variance revealed no main effect of Rating Type, $F(1,10) = 0.32$, $p > 0.6$ and only a marginal main effect of Year, $F(1,10) = 6.47$, $p = 0.064$. Critically, there was a significant interaction between Rating Type and Year, $F(1,10) = 10.08$, $p = 0.034$. As can be seen in Figure 1, as the longitudinal trend of self-ratings was stable over the three years, the *observed* ratings steadily increased from 2009 to 2011, reflecting an overall improvement in standards-based teaching practice that was independent of the participants' self-perceptions of their teaching practice.

Table 1.

Mean item scores grouped by Rating Type and Year

Rating type	2009		2010		2011	
	M	*SD*	*M*	*SD*	*M*	*SD*
Self-rated	3.9	0.6	3.9	0.6	4.0	0.4
Observed	3.3	0.5	3.8	0.5	4.3	0.5

Note. Item scores can range from 1 to 5, where 5 indicates high frequency of standards-based teaching practices.
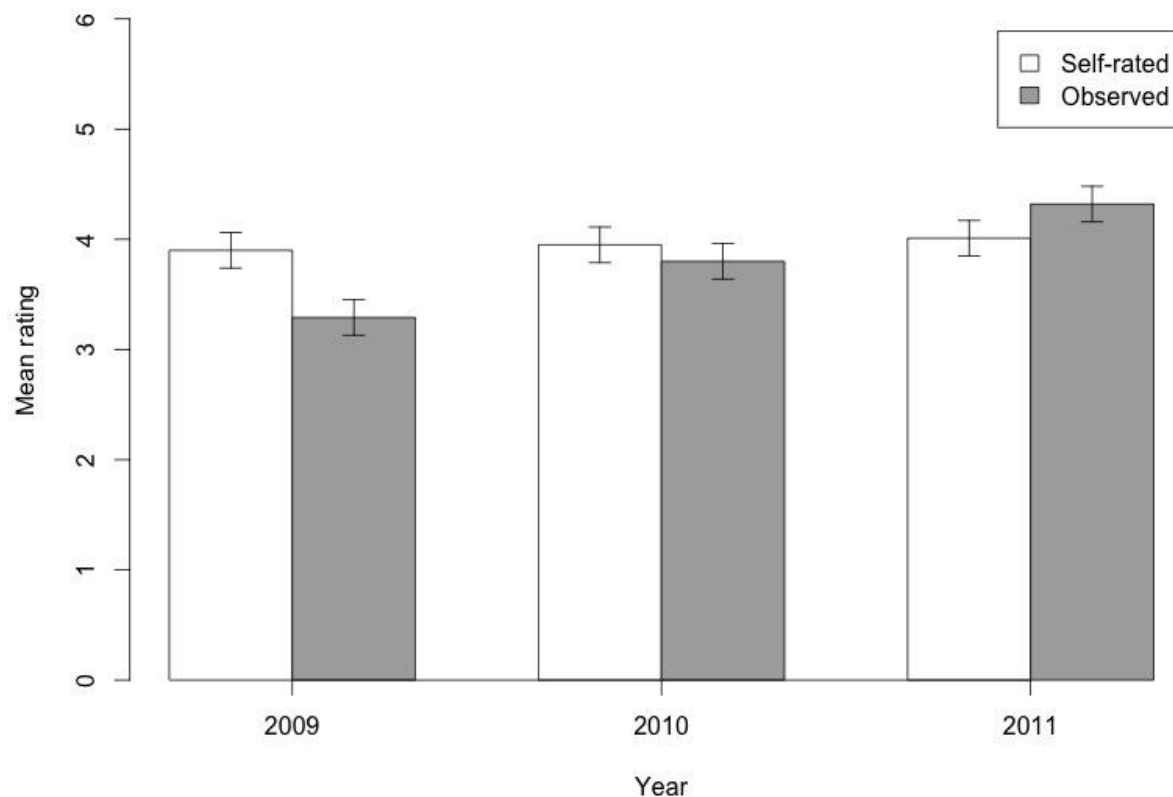
Figure 3: Mean item ratings grouped by Year and Rating Type. The error bars represent 95% confidence intervals, calculated using the method of Loftus & Masson (1994).

It is interesting to note that at the beginning of the long-term professional development project (2009), participants *over*-rated their own standards-based teaching practice a full standard deviation above the ratings of a trained outside observer. However, two years later (2011), the opposite trend prevailed in which participants *under*-rated their own teaching practice almost a full standard deviation *below* the outside rating. That is, as time progressed, participants' abilities to teach using standards-based practices increased beyond their own self-awareness. This trend is remarkable, and to our knowledge, has not been previously found in the literature on teacher development.

Discussion

The present study was designed to assess the effects of a long-term professional development program on teachers' perceptions of their abilities to teach mathematics using standards-based classroom practices. Critically, we chose to examine this through a comparison of self-reports of standards-based classroom behavior and the reports of a trained classroom observer. Predictions

based on previous literature were difficult to formulate, but we did predict a general overestimation of teachers' perceptions of their own standards-based classroom practices. Perhaps surprisingly, we did not find this. Compared to the ratings of a trained outside observer, there was no overall difference between teachers' self-rated classroom practices and the observed classroom practices.

When adding the additional variable of year, however, the data becomes more interesting. Indeed, we found a significant interaction between year and rating type. That is, as the time in the professional development program increased, teachers moved from overestimating the frequency of their standards-based classroom practices (relative to a trained outside observer) to underestimating their standards-based practices. This pattern is remarkable and was not expected.

Specifically, this interaction was due to differences in the longitudinal trends between the self-ratings and the observed ratings. Whereas self-rated standards-based classroom behaviors remained constant over the three years of the professional development program, the actual observed behaviors increased steadily. Although this growth in observed standards-based teacher practices is a testament to the efficacy of the professional development program, it is still puzzling as to why the self-ratings were stable. Two possible explanations are given below.

It is conceivable that the professional development program, by targeting specific teaching behaviors related to standards-based classroom practice, is truly effective in promoting gains in teacher efficacy while leaving teacher self-beliefs unchanged. However, data exists for this specific professional development program that calls this explanation into question. Specifically, we have seen modest, reliable gains in teacher self-efficacy over the course of the professional development program. These data are not tied to the current study, so further conclusions on this matter are left unwarranted until further studies can be conducted.

Another possibility is that over the course of professional development, teacher self-ratings are subject to a suppressive effect whereby teachers reliably underestimate their own efficacy as they become better teachers. The current data support this view but do not rule out alternative explanations, such as maturation. The case for a maturation-only account is weakened somewhat by the range of years of experience in our study (2-17 years). But nonetheless, future studies will need to carefully consider a manipulation of the professional development program, possibly by following a group of teachers who were not in such a program.

In conclusion, we examined the effects of a long-term professional development program on teachers' perceptions of their abilities to teach mathematics using standards-based classroom

practices. Through a comparison of self-reports of standards-based classroom behavior and the reports of a trained classroom observer, we found that over time, teachers shifted from a general tendency to overestimate their abilities to an underestimating tendency. The exact reason for this shift is unclear, but it is primarily due to the teachers' increased standards-based classroom behaviors through a long-term program of professional development.

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SECONDARY MATHEMATICS TEACHERS AND IWB:
PEDAGOGICAL AND PRACTICE CHANGES

Jeffrey Hall
Mercer University
Hall_JS@mercer.edu

Gregory Chamblee
Georgia Southern University
GChamblee@georgiasouthern.edu

This qualitative study explored how secondary mathematics lesson planning, classroom use, assessment methods, and student learning pedagogy and practice changed over a period of three years due to interactive whiteboard (IWB) use. Two active and two future IWB users were individually interviewed each year. Findings indicate that IWBs became increasingly vital to pedagogy, lesson planning became easier, and perceived value to student learning grew more positive for active users. Future IWB users' perceptions of the positive and negative effects remained consistent throughout the study. Implications for mathematics teacher professional development and how teachers perceive IWB-based instruction will be discussed.

The National Council for Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) Technology Principle states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). The Technology Principle also posits "technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics" (NCTM, 2000, p. 25). Today, the IWB is becoming a popular technology innovation in the mathematics classroom. Branzburg (2007) notes IWBs can be used to record lessons, data collection, present pre-made instruction (e.g. PowerPoints), allow students to move objects around the screen to see patterns, and show virtual manipulatives. Given all the possible IWB applications in the mathematics classroom, research on how teachers use this technology to enhance mathematics instruction over time, especially student learning, is needed. The purpose of this study was to explore how lesson planning, classroom use, assessments, and student learning pedagogy and practice of two active and two future mathematics teacher's IWB use changed over a period of three years.

Literature Review

The theoretical frameworks of this study were constructivism and change theory. Constructivism is a theory of learning in which learners build on previous knowledge through active participation (Piaget & Inhelder, 1966/2000). As such, constructivism encourages pedagogical strategies that promote deep conceptual understanding over rote skills (Fosnot & Perry, 2005). If interactive whiteboards are to have a positive impact on student learning of mathematics, they must enable teachers to provide conceptual lessons more effectively.

Kennewell and Beauchamp (2007) found that while some mathematics teachers take advantage of IWB capabilities, more professional development is required before IWBs fundamentally alter pedagogy. Smith, Higgins, Wall, and Miller (2005) also found positive reports of IWB usage in their review of IWB literature, but noted that it is "questionable how far the benefits reported … relate to the unique touch-sensitive nature of IWBs or merely form part of an uncritical bandwagon effect: the success of a new technology is perceived inevitable" (p. 93). More research, particularly longitudinal examinations of teacher practices, is necessary.

Change theory, according to Fullan (1993), focuses on four capacities of individuals: "personal vision-building, inquiry, mastery, and collaboration" (p. 12). Change theory is also concerned with implementation, which "focuses on what happens in practice. It is concerned with the nature and extent of actual change, as well as factors and processes that influence how and what changes are achieved" (Fullan, 1992, p. 21). Bitner and Bitner (2002) posit "before technology can effect changes in the classroom, those ultimately responsible for the classroom must be considered. Teachers must learn to use technology and must allow it to change their present teaching paradigm" (p. 95). Thus, examining teacher perceptions of interactive whiteboards as they implement the new technology is a key to understanding the change that is actually occurring in the classroom.

Time and training to develop comfort and expertise with IWBs is also important. Miller, Glover, and Averis (2005) found that "teachers need time to develop their technological fluency, apply pedagogic principles to the available materials or to the development of materials, and then to incorporate the [IWB] seamlessly into their teaching" (p. 110). Miller and Glover (2007) also note that "the introduction of technology without sufficient appropriate training in technology and teaching and learning may inhibit the realization of the full value of the equipment" (p. 329). This longitudinal study will provide some evidence of the time and training required for mathematics teachers to incorporate IWBs effectively and examine the changes that occur in their pedagogy and practices.

Methodology

The research question explored in this qualitative case study was: How do active and future secondary mathematics IWB users' lesson planning, classroom use, assessment methods, and student learning pedagogy and practice change over a three year time span? This study was conducted at a very large, suburban high school in the southeastern portion of the United States

during the academic years 2007 – 2010. Four high school mathematics teachers were selected to participate based on their experience levels with interactive whiteboards. Two teachers had an interactive whiteboard in their classrooms and were active users of the technology (Participants A & B). Two teachers did not have interactive whiteboards in their classrooms and had no experience using the technology but there were plans to place IWBs in their classrooms in the future (Participants C & D). Participants C & D did not get IWBs installed in their classrooms during the timeframe of this study. Table 1 provides additional demographic participant information.

Table 1.

Participant Information at Start of Study

Participant Name	Gender	Approximate Age	Approximate Teaching Experience	Approximate IWB Experience
A	Male	Mid 30s	10 years	1.5 years
B	Female	Mid 40s	20 years	0.5 years
C	Female	Late 30s	15 years	None
D	Male	Late 30s	15 years	None

Each teacher was individually interviewed each academic year for three years. Year One interview questions are presented in Table 2.

Table 2.

Questions for Active and Future Interactive Whiteboard (IWB) Users

#	Active IWB Users	Future IWB Users
1	"What kind of experience do you have using interactive whiteboards?"	"What kind of experience do you have using interactive whiteboards?"
2	"How have you used interactive whiteboards in your classroom?"	"How do you plan to use an interactive whiteboard in your classroom?"
3	"How have your students responded to your use of interactive whiteboards in the classroom? Please provide examples."	"How do you think your students will react to your use of an interactive whiteboard in the classroom? Please provide examples."

4	"How have interactive whiteboards changed how you plan and teach lessons? Please provide examples."	"How do you foresee the use of interactive whiteboards changing how you plan and teach lessons? Please provide examples."
5	"How have interactive whiteboards changed how you manage assessments? Please provide examples."	"How do you foresee interactive whiteboards changing how you manage assessments? Please provide examples."
6	"How have interactive whiteboards impacted student learning in your classroom? Please provide examples."	"How do you foresee interactive whiteboards impacting student learning in your classroom? Please provide examples."

Future IWB users were asked the same questions each of the three years of the study to see if their thoughts on how they would use an IWB had changed over the past academic year. Active IWB users were asked a slightly modified set of questions after the first year to determine how their pedagogy and practice had changed over the previous academic year. Data were analyzed each year for themes by group and question. Data trends across years and participants were considered at the end of the study.

Findings

The active IWB users' responses over the three years of this study indicate that pedagogy and practice do change over time. In contrast, the future IWB users' perceptions and expectations remained consistent over the duration of the study. Findings will be presented by participant and then overall data trends will be discussed.

In Year One, Participant A stated that he was trying to use his IWB in a couple of novel ways. First, he stated that he was trying to record his lessons for future playback for students who were absent, although he criticized the significant amount of time required to do so. By the third interview, Participant A was less interested in this strategy of recording lessons. Instead, he was now collaborating and sharing IWB material with two other teachers. These materials could also be shared with absent students, which effectively eliminated the need to record lessons. Participant A also reported in the first interview that he was trying to incorporate the TI-Navigator system (which networks a class set of graphing calculators) in order to display individual student calculator screens on his IWB. This technology was intriguing to Participant A, but he felt it was prone to technical difficulties. By the third interview, Participant A was using the TI-Navigator system less often, and instead projected a virtual TI-84 calculator on his

IWB screen when he needed to incorporate graphing calculators into his lessons. While this approach eliminated the network advantages offered by the TI-Navigator system, it was easier to manage in an actual classroom setting. He also began using the ActiVote system that comes with Promethean-branded IWBs, allowing students to register answers to IWB-based questions from their seats.

Participant B, who was a very new IWB user at the time of the first interview, demonstrated different changes over time. Initially, she used her IWB to primarily project PowerPoint slides and mathematics worksheets on the screen. In order to take more advantage of unique IWB capabilities, she projected a virtual TI-83 graphing calculator that she could manipulate by touch. She also tried to save IWB-based lessons for absent students that she would disseminate via her teacher webpage, but file sizes were too large to upload, so she instructed absent students to bring in personal flash drives instead. Since she was concerned that students would damage the IWB, she did not allow them to personally interact with the technology very much.

By the third interview, Participant B had expanded her IWB use significantly. Instead of relying on PowerPoint slides, she had started teaching with an online textbook that displayed pages identical to her students' textbooks. She also sought online manipulatives to teach mathematical concepts, like a "Plinko" simulator to demonstrate normal distributions. Over time, she had lost her fear of students damaging the IWB, and now actively encouraged her students to interact with the board. She also incorporated drop-down screens in order to display information when needed on the IWB, an effect that she felt made her presentations more dramatic and visually appealing.

However, IWBs did not always provide the preferred solution for Participant B. For example, she had tried to use her IWB to project questions during an assessment, but students complained that they could not see the board well enough to take the quiz. Also, in a step away from the touch-screen capability of the IWB, Participant B had purchased a wireless tablet input device with her own funds in order to control the computer remotely. This enabled her to stand in the back of the room and still manipulate what students saw on the screen, allowing her to pace around the room and not have her back to her students.

Both Participants A and B had attended a brief training course in the summer prior to receiving their IWBs, but learned informally on their own or in collaboration with other teachers after the initial training. In their opinion, the initial training was sufficient to get them started.

Their own interest in utilizing the IWB propelled them to try and learn new methods of incorporating the technology.

In contrast to the changing views of the current IWB users, the two future IWB users in this study provided very consistent responses over the course of three years. Both Participants C and D had attended a brief workshop introducing IWB capabilities prior to the first year interview, so they were familiar with the technology, but had never actually used it themselves. Participant C anticipated using the TI-Navigator system in conjunction with her IWB. She also wanted to encourage student interaction by allowing her students to manipulate the IWB themselves. In contrast, Participant D taught statistics, so he was interested in displaying statistical information from the internet to enrich his lessons. He also wanted to display statistical programs (such as Fathom) for students to see while solving problems. These responses remained remarkably consistent over the course of the three interviews, indicating that change in perception will not occur without training or active usage of the technology.

Based on the interview responses over three years, it is clear that IWBs can become indispensible to teachers, even if they do not always maximize its capabilities. As their experience increased and the time required preparing IWB-based lessons decreased, both participants indicated that they would not want to teach without an IWB anymore. They continued to explore new ways to command student attention and teach mathematical concepts, but were unsure if student learning is being impacted. In fact, Participant B stated that traditional homework was still the primary means of ensuring student learning. From an assessment perspective, IWBs appear to hold less promise, although Participant A's experiments with the ActiVote system might provide an effective method of gauging student understanding.

Summary

Branzburg's (2007) concepts of IWB usage were noted by future users as great classroom ideas. The active IWB users implemented many of the suggested uses. Participants A & B did change their uses and perceptions of IWBs regarding lesson planning and classroom practices, but assessment methods were mostly unchanged by the technology due to logistical concerns. Participants A &B also discussed how they used what they learned from their previous experiences to implement new IWB uses. Participants C & D did not change very much in their thought processes. Data trends from this study support change theory and the constructivism

theories regarding how individuals learn and change their thoughts. Therefore, using these two frameworks to assess technological change over time is encouraged by this study.

Similar to the findings of Glover, Miller, and Averis (2005), the results of this study support the importance of time for teachers to effectively integrate IWBs into their daily classroom routines. The active IWB users indicated substantial changes in their usage in the third year compared to the first year. However, training with IWBs does not appear to be necessary after an initial introduction to the technology. Both active IWB users indicated that a brief workshop was all they needed to get started, and developed expertise either on their own or by collaborating with other IWB users. While long term training may be required for some teachers as noted by Glover and Miller (2007), the active users in this study did not perceive a need for it.

Based on the active IWB users' responses, the effects of IWB use on student learning are perceived to be positive, but the issue requires further study. As teachers learn to take greater advantage of unique IWB capabilities, their perceptions of student learning should improve. The researchers recommend that longer case studies of teachers may be needed to see this change. A quantitative examination of student learning due to effective IWB use would also be vital.

Finally, the results of this longitudinal study support Smith et al.'s (2005) findings that teachers perceive IWBs as offering unique capabilities that are beneficial to both students and themselves. This conclusion even applies to teachers who have not actively used the technology, but are familiar with its capabilities. As a result of these positive perceptions, teachers will voluntarily try new methods of teaching and assessing students in search of best practices. With experience, training, and collaboration, new pedagogical strategies will evolve to take advantage of the IWB capabilities.

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PATTERNS OF PARTICIPATION AMONG SECONDARY TEACHERS IN A MASTER OF SCIENCE IN MATHEMATICS PROGRAM

Elaine Young
Texas A&M University - Corpus Christi
elaine.young@tamucc.edu

Joe Champion
Texas A&M University - Corpus Christi
joe.champion@tamucc.edu

Sarah Ives
Texas A&M University - Corpus Christi
sarah.ives@tamucc.edu

George Tintera
Texas A&M University - Corpus Christi
george.tintera@tamucc.edu

This study identifies participation outcomes of 118 secondary mathematics teachers in a MS Math program at a mid-sized university in the southern U.S. Grant funds supported cohorts of 30 inservice secondary mathematics teachers in taking one graduate mathematics course per semester. Data sources included demographics, teaching observations, and employment records. Mixed methods findings included implications of four participation patterns: (1) transition to teacher leadership, (2) developing competency in mathematical knowledge, (3) sporadic progress, and (4) plateauing at the culminating master's project.

Improving the specialized content knowledge needed by inservice mathematics teachers is an important function of mathematics education (Ball, Thames, & Phelps, 2008). Nearly half of U.S. public school teachers have earned a master's degree (Hill, 2007) and the demand for master's programs among inservice teachers remains strong (Roza & Miller, 2009). Teacher participation in master's programs in mathematics and education are both potentially valuable because the two program types offer complimentary perspectives on the field of mathematics. However, mathematics teachers increasingly have chosen to complete master's degrees in education. In 1970, the ratio of master's degrees in education to degrees in mathematics was more than 16:1. By 2009, that ratio was 35:1, with less than 1% annual increase in master's degrees in mathematics compared to 104% increase in education (NCES, 2010). This study considers one university's approach to increased participation in graduate mathematics courses by recruiting and financially supporting inservice mathematics teachers in an M.S. Math program.

Since 2004, mathematics faculty at the research site have delivered a series of 1- to 3-year grant cycles supporting cohorts of up to 30 inservice mathematics teachers through graduate mathematics courses with special emphasis on connections to content and process standards from the National Council of Teachers of Mathematics (NCTM, 2000). The grant program, funded by federal flow-through grants and coordinated by state education agencies, have financially

supported a total of 138 teachers through 4 grant cycles over 7 years by covering costs such as tuition, fees, textbooks, materials, childcare, mileage, conference travel, course stipends and technology tools. Grant participants typically enrolled in one of the specially designed graduate mathematics course per semester. Grant courses taught mathematics educators in the mathematics department included courses such as *Structure of Patterns & Algebra*, *Structure of Geometry & Measurement*, *Mathematics Assessment*, *Evolution of Mathematical Systems*, and *Problem Solving & Mathematical Reasoning for Teachers*.

The main goal of the grant programs has always been to increase the mathematical content knowledge of inservice teachers with non-traditional certifications (emergency, probationary, or alternative) or who are teaching out of the content field (no major/minor in mathematics or have not passed state mathematics certification tests). An aim of the granting agency was to prepare middle school teachers with at least 30 college math credits and 12 mathematical pedagogy credits and high school teachers with 36 college math credits and 12 pedagogy credits. Despite the special emphasis of courses on standards-based practices, there are inherent challenges in offering graduate mathematics courses to inservice teachers who may have limited undergraduate mathematics experience as well as implications of limited mathematics background on program retention and completion outcomes.

This study focuses on a single research question: How can patterns of participation help to clarify the consequences of offering graduate mathematics courses to diversely prepared secondary mathematics teachers? The study narrative focuses on qualitative patterns of participation of teachers in the grant programs with the aim of illustrating the varied professional contexts of program participants. Some of the changes to the program in response to this study are also indicated.

Methodology

Data Sources and Analysis

Research and evaluation of the grant programs has stimulated extensive data collection throughout the seven-year study period, including gender, ethnicity, undergraduate and graduate transcripts, current teaching positions, certification records, and date of birth. This, combined with course work and teaching observations by program faculty, provided detailed information on the teachers' professional responsibilities and work contexts.

Due to the many types of data available on program participants, data analysis focused on the

qualitative practices of conceptualizing and reducing data to descriptive accounts, elaborating on emerging themes, and drawing relations among participants' experiences (Strauss & Corbin, 1998). This, in combination with detailed statistical summaries, allowed the development of composite profiles of participants' participation in the grant program in the form of a collective case study (Creswell, 2007) with special emphasis on sampling with maximum variation across participants' experiences (Patton, 2002).

Participants

Of the 138 participating teachers across four grant cycles, complete data exists for $N = 118$. The demographic profile reported 71% Female, 29% Male, 48% Hispanic, 42% White, 7% African-American, and 3% Asian-American. The mean age at the start of participation was 38 years old ($SD = 11$, Range = 23 to 75). Teachers' undergraduate preparation varied widely, with transcript totals averaging 21 semester credits ($SD = 14$, Range = 0 to 64) in mathematics and 2 semester credits ($SD = 4$, Range = 0 to 15) in mathematical pedagogy courses (two-thirds teachers had completed no pedagogy courses). Almost a quarter (24%) held a bachelor's degree in mathematics; others held degrees in education (29%), science or engineering (15%), sports and health sciences, (13%), business (11%), and liberal arts (7%). Twenty-seven participants (21%) had previously completed a master's degree in education (20), business (3), science (3), or liberal arts (1).

Though all participants joined the program while teaching at least one math course by assignment, 40% had not completed a teaching certificate prior to joining the program. Many of these completed their certification during the grant program. Certified teachers joined the program an average of 5 years ($SD = 9$, Range = 0 to 41) after obtaining certification. Teachers were employed at 18 different districts and several private, parochial or charter schools, with 91% teaching in public schools. There are no urban districts in the region, and 82% of most teachers taught in a suburban school. Approximately equal groups of teachers taught at the middle and high school levels (49% high school, 45% middle school, 6% both levels over the grant program). At the beginning of their grant participation, 42% of the teachers were initially admitted to the MS in Mathematics degree program, 31% to a master's program in the college of education, 10% to non-degree graduate status, and 17% participated for professional development hours only (no college credit). This last category was required by the grant as an option for participants.

An important basic outcome of the teachers' participation in the grant programs has been graduate program retention and completion. About 23% of the 118 teachers in the study are participating in the current grant cycle, 19% have completed a master's degree, and the remaining teachers (58%) ended participation short of earning a graduate degree. There are important implications and insights for each of these three groups, and the qualitative analysis has led to thematic composite profiles within the groups to illustrate the rich variety of teachers' participation patterns. Throughout the findings, eight composite profiles (under pseudonyms) appear in text boxes with descriptive characteristics most typical of teachers in those respective groups. Figure 1 shows the eight profiles within the context of program status and in proportion to the number of teachers represented by each profile.

Figure 1. Composite profiles in proportion to number of teachers represented by each profile. Unmarked light blue and light green regions indicate 14 teachers not represented in the profiles.

Current Participants

There are twenty-seven teachers currently participating in the grant program; nine new recruits started within the past six months and three teachers are nearing graduation. One participation pattern that emerged from the current cohort is that of start-stop-start again. The data show two different trends in this subgroup of teachers.

Nancy is a 41-year old White middle-school teacher. She began her teaching career with probationary certification and completed certification in 2007. She began the grant program in 2008 with 8 college math classes and 1 pedagogy class. She has completed 5 of 12 grant classes over a span of 4 years. Her fluctuating participation in the grant program is due to personal issues or distance from the university.

Elena is a 60-year old Hispanic high-school mathematics teacher. She was certified in 1990 before the current certification standards. She began the grant program in 2009 with 7 college math classes and no pedagogy classes. She has completed 3 of 6 grant classes over a span of 2 years. Her fluctuating participation in the grant program is due to personal issues.

Program Graduates

Twenty-three participating teachers have graduated with a master's degree in mathematics ($n = 19$) or education ($n = 4$). Six of these entered the grant program with a prior master's degree in education, business, or computer science. Teachers earning an education masters took three to six grant courses as electives in their education degree program. Teachers earning mathematics degrees took six to eleven grant courses. One teacher transitioned to post-secondary teaching (making her ineligible for grant support) and completed the graduate mathematics program at her own expense.

Two general patterns were noticed among the graduates. Eight teachers came into the master's program with low credits in mathematics, yet graduated at a high level of ability and have been helping to make changes in their schools. Seven teachers began the master's program with high credits in mathematics, and moved into school leadership positions and mentoring roles during the program and after graduation. The data show two trends in the graduating teacher group.

Sharon is a 47-year old White high-school mathematics teacher who started her career with probationary certification. She was more likely to have a bachelor's degree in liberal arts or science than in education. She averaged 16 college math credits and no pedagogy courses at the beginning of grant participation. She went on to become a math department chair, a math coach, or a support person for mathematics at the district level.

> Luisa is a 39-year old Hispanic high-school mathematics teacher who entered the program with more than 24 college math credits and 3 pedagogy credits. She took 6 to 12 grant courses while completing the master's program. She actively took what she learned in the grant classes back to her school and mentored her colleagues. She then rose to hold a leadership position in her department, completed national certification, or became a mathematics specialist at the district level.

Participation Stopped Short of a Master's Degree

Sixty-eight teachers stopped short of completing a graduate program. These teachers can be partitioned into several subcategories: (a) moved out of the area, (b) did not finish the first class, (c) did not finish the grant cycle, (d) stopped participating in the grant program after initial success, or (e) worked successfully before hitting a plateau at the end of the graduate program.

Did Not Finish the First Class

Seven teachers were unable to complete their first grant class due to various reasons. This group of teachers spanned the entire seven years of the study.

> Penny is a 45-year old White middle school mathematics teacher in her fourth year of teaching. She entered the grant program seeking professional development hours rather than college credit. She began with less than 3 college math credits and no pedagogy credits. She withdrew or took an incomplete for the first grant course due to personal or family health issues, work concerns, or difficulties keeping up with classwork.

Did Not Finish the Grant Cycle

Thirty-six teachers did not finish the grant cycle (most grant cycles were one-year long). Seventeen teachers only finished one class; nineteen finished two classes. At least seven of these teachers are no longer teaching; either they did not complete certification or they burned out of teaching and changed careers. Nine of these teachers did eventually complete a master's degree in education.

> Hilda is a 41-year old Hispanic middle-school mathematics teacher who was enrolled in an education master's program. She was most likely certified in mathematics but may have certified in other fields, or have no certification. She began the grant program with five college math classes and one pedagogy class. She completed 1-2 grant courses, withdrawing short of completing the grant cycle.

Stopped Participating after Initial Success

Thirteen teachers stopped participating in the grant program after successfully completing

three to seven grant classes. Five teachers completed a one-year grant cycle (three classes). Eight teachers completed four to seven classes before stopping (it is interesting to note that all eight were Hispanic). Two teachers had expired probationary certificates due to their inability to pass the certification test.

> Pablo is a 43-year old Hispanic high-school mathematics teacher who was enrolled in an education master's program. He started his teaching career on probationary, emergency, or alternative certification. He entered the grant program with 8 college math classes and 3 pedagogy credits. His undergraduate degree is most likely not mathematics or education, but may be business, history, or science.

Plateaued after Sustained Success in Courses

Six participants hit a plateau in their master's program. After completing all core classes and electives, they were unable to complete the master's project. They started in the grant program between 2004 and 2006, and most had completely stopped taking classes by 2008. Some of these teachers were displaced from teaching jobs multiple times, took time off of teaching and then returned to the classroom, moved to teaching at the elementary school level, or stopped teaching completely.

> Polly is a 42-year old White middle-school mathematics teacher who was enrolled in the mathematics master's program. She began the grant program with 24 college math credits and 3 pedagogy credits. She was as likely to have started teaching on a probationary certificate as not. She completed ten grant classes, and then faltered during the graduate project phase. She took the project proposal and/or project class up to eight times, but was unable to finish the master's project and graduate.

Discussion

The results illustrate several consequences of offering mathematics graduate courses to inservice secondary teachers with diverse mathematical backgrounds. While it's difficult to draw conclusions regarding current teachers in the program, many of the teachers in both the "start-stop-start" and "stopped after limited coursework" patterns reported external and personal reasons for rethinking program participation. For example, 20 of 23 graduating teachers lived within an hour of campus, while only 2 of 9 teachers who exhibited the start-stop-start pattern lived within an hour. Considering the barrier of long-distance travel has been an emerging goal

of program faculty, and the program has begun to consider hybrid courses with virtual classes on some weeks.

The 23 teachers who graduated the program suggest that success in specially-designed graduate mathematics is possible for inservice teachers with diverse mathematical backgrounds. From Sharon (comparatively weak mathematics background) to Louisa (strong mathematics background), many teachers exited the program with high mathematical abilities and went on to become leaders and mentors in their schools and districts. This suggests the grant program has afforded opportunities for numerous teachers to leverage mathematics courses as part of their professional growth, and the program faculty have leveraged that success by involving successful teachers as informal program mentors and adjunct instructors at the university.

Program faculty have been particularly concerned about the pattern of plateaued participation. Inservice teachers who complete all but one or two courses in a graduate mathematics program represent large financial and time investments on the part of both teachers and faculty. One approach addressing this barrier to graduation has been to develop broader yet more focused guidelines for the culminating master's project, including suggestions for building on existing master's projects and forming collaboratives among teachers with similar interests or work contexts. Another approach has been to increase writing into core classes and electives to better prepare teachers to write a graduate project proposal and manuscript. A third response to this study has faculty working to recruit program participants through school teams with the goal of developing school-based collaboratives which can build success amongst groups of teachers.

Limitations

Readers of this report are cautioned to carefully consider the research methodology and context-dependent aspects of the findings before drawing connections between these results and their graduate programs. The study period was prolonged and data collection was nearly comprehensive. The transferability of findings is limited by possible large differences across institutions regarding the personal, academic, and professional backgrounds of inservice teachers who participate in graduate mathematics courses, grant or financial support structures, and faculty experience and philosophy. Institutions offering graduate mathematics courses to inservice teachers are encouraged to consider the commonalities and differences between these findings and patterns of participation at their individual institutions.

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TEACHERS' CLASSROOM PRACTICES USING TECHNOLOGY FOR FORMATIVE ASSESSMENT: THE CONNECTION TO KNOWLEDGE AND PERCEPTIONS OF MATHEMATICS, LEARNING AND TEACHING

Hannah Slovin	Judith Olson	Melfried Olson
University of Hawai'i	University of Hawai'i	University of Hawai'i
hslovin@hawaii.edu	jkolson@hawaii.edu	melfried@hawaii.edu

We report on case studies conducted with six seventh-grade mathematics teachers who participated in a two-year professional development research study on implementing formative assessment in networked classrooms. The report focuses on data from the semi-structured interviews conducted at the end of the project. We describe three of the categories—formative assessment, pedagogy, and student role—that emerged from the coding (Strauss & Corbin, 1998) and relate these to teachers' use of the technology.

There are few studies that examine teacher learning and practice for more than a year, especially looking at the impact of technology-focused professional development (PD) (Mouza, 2006). In this paper, the impact of two years of PD on teachers' implementation of formative assessment in a connected classroom is analyzed through the lens of the interactive relationship between practices and beliefs. Analysis is based on case study data collected from six of 30 teachers who participated in Project FANC[1], a research study of implementing formative assessment in a networked classroom using the TI-Navigator System[2] and graphing calculators. The case studies are part of a larger investigation comparing two models of PD delivery on the use of formative assessment in a networked classroom as it affects middle grades student learning of algebra concepts. (For more information about the FANC Project research and results, see Yin, Brandon, Olson, Slovin, & Olson, 2011.) The purpose of the case studies was to obtain a thorough and nuanced understanding of the manner in which the teachers implemented formative assessment and TI-Navigator after participating in professional development. Our analysis aimed at providing rich accounts of the context of conducting formative assessment, the differences among teachers in their use of TI-Navigator for formative assessment, and how the PD affected other factors related to teachers' practice.

The FANC Project was designed to address a major hurdle in utilizing formative classroom assessment related to the collection, management and analysis of data. Feedback loops can be very slow, making it difficult for teachers to use formative assessment information in a timely manner. Classroom networked technology has the capability to provide rapid cycles of feedback to improve ongoing activity in real time (Roschelle, Penuel, & Abrahamson, 2004). The TI-

Navigator System's four functions support formative assessment: (a) Quick Poll—allowing teachers to instantly collect and display all students' responses to a question; (b) Screen Capture—allowing teachers to monitor individual students' progress at anytime; (c) Learn Check—allowing teachers to administer quick and frequent check-ups and provide timely feedback; and (d) Activity Center—allowing students to work collaboratively to contribute individual data to a class activity.

The connection, however, between obtaining information about students' knowledge and implementing changes in instructional practice is not automatic. Researchers at Ohio State University found stronger evidence for technology implementation than for change in instruction. Even with technology tools available to assist with implementation of formative assessment instructional strategies, not all teachers who increased their use of technology necessarily made full use of the potential of the connected classroom for formative assessment (Owens, et al., 2008).

Substantial evidence suggests there is a complex relationship among teachers' classroom practices and the interconnected knowledge and perceptions of mathematics, technology, learning, and teaching. The integration of technology in classroom practices is influenced by teachers' background knowledge and experiences, and conceptions of technology, mathematics, and learning (Zbiek & Hollebrands, 2008). Teachers' disposition towards mathematics and its pedagogies is another major factor in determining how they make sense of technological tools and their integration in classrooms (Ruthven, Deaney, & Hennessy, 2004; Mousley, Lambdin, & Koc, 2003). Mouza (2006) found that change in teachers' use of technology was highly dependent on the continual interaction between practices and beliefs, including beliefs about students, and that teachers fluctuated between using practices learned in PD and returning to former practices.

Methodology

Ten teachers were included in the case studies, five from each of the two PD models. The sample was chosen to represent the range of prior familiarity with formative assessment as determined by surveys and from schools located in the variety of communities representative of student populations in Hawai'i's public schools. All ten teachers were part of the case studies for the two years of the project.

From the 30 participating teachers, data were collected regarding mathematical content knowledge for teaching, responses to writing prompts, beliefs and perceptions about mathematics teaching and learning, efficacy in using formative assessment and technology, and support within the school community. In addition, case study data included more complete background information provided by teachers, classroom observation field notes (four or five observations per teacher over two years), notes from coaching visits (two or three per year), focus groups and individual interview videos. The researchers who conducted the case studies were not involved in PD delivery or in coaching visits. Our report focuses on issues of implementation of formative assessment in a networked classroom and not on individual teachers.

While case study data were collected for 10 participants, this paper concentrates on the interview data from six of the teachers. These six were chosen because they represented a wide range of uses of technology and differed in their definitions and implementation of formative assessment. The one male and five female teachers ranged in age from their late 20s to early 40s. Their teaching experience ranged from one year to eleven years. Four of the teachers had a degree in secondary mathematics education and were certified to teach secondary mathematics and one teacher had certification to teach elementary and middle school. One teacher with a degree in communication and computing was a Teach for America graduate who obtained her M.Ed. and was certified in mathematics by the end of her participation in Project FANC. One of the teachers in this study had received National Board Certification.

The six teachers came from a representative group of schools with respect to ethnicities in Hawai'i, socio-economic status, and from urban and rural locations. Technology available in the schools and prior experience with technology varied among the teachers.

Results

In this report on the semi-structured interviews conducted at the end of the FANC Project, we focus on three of the categories that emerged from the data analysis (see Olson, Gilbert, Slovin, Olson, & La, 2011). These categories involve teachers' views of formative assessment, pedagogy, and the role of students. In the complexity of classroom life, we believe that these aspects are interrelated in numerous ways, however, we discuss each category separately before looking at their role in teachers' use of technology in the classroom.

Formative Assessment

All teachers in the case study had some familiarity with formative assessment before entering the FANC project. Some had participated in PD sessions about formative assessment given by the state department of education. We highlight two themes: 1) the variation in teachers' views about formative assessment and 2) whether teachers reported on having changed their views of formative assessment as a result of their participation in FANC.

While all teachers believed formative assessment yielded valuable information, their views ranged from conceiving of formative assessment as a series of 'pulse-checks' to a view of formative assessment tasks indistinguishable from tasks of the ongoing lesson. At one end of the spectrum, teachers used questions and check-ups as one would use instant quizzes to monitor students. Information from students' responses enabled them to know who was following the lesson, which students understood, and as guidance for pacing. These teachers tended to use Quick Poll and Learn Check more than any other of the Navigator features. Moreover, they interpreted students' responses in terms of correct or incorrect answers. Denby's comment represents this viewpoint. "I use it more just…to assess, Where are they? How many of them got this? Can I move on? Or do I have to still wait and go back and check?"

Other teachers used students' responses to Quick Polls and Learn Checks to focus on misconceptions. Displaying these results provided opportunities for the whole class to discuss misunderstandings. Such discussions helped teachers better understand student thinking, and by making incorrect answers public, students and teacher were able to jointly learn from and address incorrect responses.

In Clarise's classroom, formative assessment was woven into all teaching and learning activities. Since Clarise had changed her grading policies to exclude assignments such as homework, she says, "Homework…is more like a discussion point rather than something to grade." (Clarise) In this way, the homework responses, collected on Quick Poll, became part of the dynamic of teaching and learning, so that both students and teacher could formatively assess understanding in the course of the lesson.

Almost all teachers reported that their participation in the project had broadened their view of formative assessment. Instead of distinguishing between formative and summative assessment based on the amount of material covered, as most teachers previously had done, they reported

realizing that there are many formats for conducting formative assessment, and that formative assessment can be a daily occurrence. Yaz expressed his new understanding,

> …formative assessment is more than just a quiz and seeing where kids are and what I need to re-teach. It goes into depth on misconceptions and why kids think this way, how can we address it…. (Yaz)

Pedagogy

Among the pedagogical issues that arose in the interviews, two key topics were questioning and planning. Both of these are important aspects of formative assessment (Ayala & Brandon, 2008) and both are integral to using the Navigator features. In their interviews, most teachers focused on the importance of questioning, but not all used questions in the same way. Iris was representative of teachers who used questions to guide students.

> I try to direct them in the way I want them to go, so to speak. With the equations, where they had to put their own equations, I ask them, "What is the coefficient?" And using that vocabulary, trying to get them used to the vocabulary…. (Iris)

Other teachers tried to probe students' thinking through questioning. They created questions to expose misconceptions so they could provide interventions as needed. They spoke about "thinking as a student yourself" (Yaz) and the importance, when introducing new concepts, of knowing, "what might they have a hard time with, what might they misunderstand or misinterpret…" (Kate). Kate distinguishes between questioning and telling.

> I like to ask kids questions, rather than direct them to an answer or tell them their answer is correct. I like to know more about why they think they are right or wrong…. (Kate)

Teachers varied in how they planned lessons. The teachers who were focused on student thinking tried to anticipate what problems and misunderstandings students would have. Yaz and Kate, who collaborate in their planning, like to have students' misconceptions in mind as they plan, "…then we are ready for the discussion…." (Kate). Clarise also focused on students' thinking. In her efforts to be responsive, her approach to teaching has become more spontaneous. "…Sometimes it's [referring to a Quick Poll question] planned ahead of time, but most of the time it's just right in the moment…." (Clarise)

By contrast, Denby's goal in planning was to create a lesson that maximized students successfully completing the tasks she intended for them. Her planning strategy was to carefully break down tasks into smaller pieces so she could assess if students understood before moving

on. She used this approach when introducing the Navigator technology and for introducing mathematics content. Denby believed this approach supported learning goals and the effectiveness of lessons.

Student Role

The TI-Navigator system is connected to a projector enabling everyone to see student responses. While it is possible for these responses to be anonymous, students usually know and identify which response belongs to them. Teachers commented on the public nature of the display.

> With the Navigator, it holds them a little more accountable. 'Cause they know if they are the one student who is not responding, …the one the class is waiting for…. With the "Are You On the Line?," they know if they are the one on or off the line because they weren't paying attention. …so, it holds them a lot more accountable to participate. To be engaged. (Kate)

> …having the Navigator…they put in the equation and [they] could immediately see who is getting it right or wrong, and we could help them make the corrections right away. (Iris)

The Navigator features also enable students to provide feedback for others. In this way, students become sources of knowledge for the class and thereby assume some of the responsibility for the group's learning, both to prompt thinking about the tasks and to assess understanding. "My kids have picked up on my questioning… they make their peers think about it…." (Kate) Not all teachers promoted a shared locus of control to the same degree, but all teachers reported on noticing and using an increase in student discourse to guide their teaching.

> …it's a whole lot better when they're talking with each other. They teach each other stuff, and then there [are] ten teachers in the room instead of just one. (Clarise)

Discussion: Integration of Technology

Prior to the study, none of the teachers had used TI-Navigator, and they varied in their overall experiences with technology. So it is not surprising that over the course of the FANC project, teachers used the Navigator system in a variety of ways in their lessons. Some examples include using Quick Poll or Learn Check at the beginning of a lesson as a warm up; posing problems in Activity Center to which all students contributed data; and displaying student work on Screen Capture so students could compare work in progress. All teachers used the various

Navigator features to get feedback about students' progress at critical points during the lesson. Most teachers used Quick Poll and Learn Check more often than Activity Center or Screen Capture. They reported taking longer to become comfortable with Activity Center, and they viewed its applicability mainly to algebra topics, especially graphing of data or exploring features of equations.

Teachers believed that the immediate feedback from all of the Navigator features significantly supported teaching and learning in their classrooms. Yet, there was a subtle difference in viewpoint about how that support was utilized that relates to their approaches to formative assessment, pedagogy and the role of their students. Some teachers primarily used the feedback as assessment for how the students were doing. These tended to be the same teachers who viewed formative assessment as measures of correctness or incorrectness, whose questioning was designed to direct the lesson, and who viewed students primarily as receivers of information.

> Class, why do you think so-and-so got this answer? Oh, because they did the wrong step…they multiply first instead of doing power first, whatever…So that's very valuable…you have instantaneous, it's formative assessment and I can check it and see who's got [it] and who doesn't; that's huge. (Sharon)

Another group of teachers viewed the use of the Navigator system as affording greater learning opportunities for their students. These teachers also utilized the feedback from student responses to inform their teaching as the lesson activity progressed.

> When…we are going over the answers and I see two different proportions set-ups…as being correct, I would use that as a discussion on why these two look different…and are still…considered… correct…. And then we had a discussion with the kids. (Yaz)

In summary, were we able to determine different levels of use of technology and formative assessment, and we found this use corresponded to teachers' beliefs and perceptions related to their views of formative assessment, classroom pedagogy, and students' role in the teaching and learning process. These views then had an impact on how they used technology for implementing formative assessment in a networked classroom.

In half of the cases, teachers commented that they altered their initial beliefs and practices as a result of new knowledge and experiences acquired through FANC PD experiences. This is referred to as transformative learning (Mezirow, 1997). Clarise, Yaz and Kate discussed how

their new knowledge of using TI-Navigator for formative assessment changed the manner in which they planned and carried out their instruction. Other teachers melded new beliefs into existing ones or substantiated previously held points of view which is referred to as additive learning, (Thompson & Zeuli,, 1999). Denby, Iris and Sharon made comments that would indicate that they viewed the use of TI-Navigator for formative assessment as a means of adding to their existing practice of checking on students' correctness of answers. In this study, similar to other research, beliefs influenced practice and became influenced by the implementation of new technology (Mouza, 2006).

Endnotes

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2. TI-Navigator™ is a networking system developed by Texas Instruments that wirelessly connects each student's graphing calculator to a classroom computer.

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MATHEMATICS INSTRUCTIONAL QUALITY AND STUDENTS' OPPORTUNITIES TO LEARN MATHEMATICS

Temple A. Walkowiak
North Carolina State University
temple_walkowiak@ncsu.edu

Robert Q. Berry
University of Virginia
robertberry@virginia.edu

This study used observational methodology to examine and describe the differences in mathematics instructional quality between two groups of teachers. Through two cases of teachers based on their mathematical knowledge for teaching and mathematics teaching efficacy, the findings show the differences in students' opportunities to learn mathematics.

Research has indicated that differences in the quality of mathematics instruction among teachers account for substantial differences in student mathematics achievement (Gordon, Kane, & Staiger, 2006; Rivkin, Hanushek, & Kain, 2005). There are concerns that high-quality mathematics instruction is not the norm for all students. Teachers first need an understanding of its components as outlined by the National Council of Teachers of Mathematics (NCTM). These components include providing relevant and engaging *tasks* for students, promoting *discourse* among students, and creating a *mathematical learning environment* that allows for sense-making of mathematical ideas (NCTM, 2007). There is a disparity between NCTM's vision and the nature of actual mathematics instruction in many U.S. classrooms (Hiebert et al., 2005). Researchers have suggested this stems from inadequate research focusing on mathematics instructional quality and the specific kinds of pedagogical moves that take place during high-quality mathematics teaching (Clements, 2007). Providing specific examples and non-examples of high-quality mathematics instruction requires an examination of current practices among teachers. The current study addresses this need; its purpose is to compare mathematics instruction of elementary teachers with above versus below average mathematical knowledge for teaching (MKT) and mathematics teaching efficacy (MTE).

Related Literature

The constructs of interest, MKT and MTE, are two factors that likely impact instructional practice. Ball, Thames, and Phelps (2008) have defined MKT as the "common content knowledge" needed in everyday life plus the special type of knowledge only a teacher needs. Some of this special knowledge includes interpreting solution strategies, being familiar with common misconceptions among students, and choosing appropriate representations. Research has identified a unique contribution of a teacher's MKT to student achievement in mathematics

(Hill, Rowan, & Ball, 2005). Additionally, scholars have found positive links between a teacher's MKT and the mathematics instructional quality he/she offers to students (Sowder, Philipp, Armstrong, & Schappelle, 1998).

The second construct, mathematics teaching efficacy (MTE), is based on researchers' conceptualization of general teacher efficacy (Dembo & Gibson, 1985). MTE includes a teacher's belief in his or her abilities to be an effective mathematics teacher and in how much he or she can impact learning regardless of students' backgrounds (Enochs et al., 2000). High efficacy among teachers has been linked to teaching approaches that are consistent with standards-based instruction outlined by NCTM (Haney, Czerniak, & Lumpe, 1996).

Examining the instructional practices of teachers with both high or low MKT *and* MTE is absent from the literature. The current study addresses this gap. Further, it responds to the lack of research describing examples and non-examples of specific pedagogical moves associated with high-quality mathematics instruction. The research question guiding this study is: What are the similarities and differences during mathematics instruction in the *tasks*, *discourse*, and *mathematical learning environment for sense-making* between teachers of above average and below average MKT and MTE?

Methodology

The work of this study comes from a larger one of 103 third-grade teachers. A latent profile analysis determined the predominant profiles of teachers based on each teacher's mathematics instructional quality, MKT, and MTE, all assessed via quantitative measures (Walkowiak, 2010). The current study focuses on four teachers in that sample who were purposefully selected because they had the highest probability of belonging to their given profile.

A case study, qualitative research design was employed in this study. The first case describes Profile One through the classrooms of two teachers with above average MKT and MTE scores, relative to the sample of 103 third-grade teachers. The second case details Profile Two through two teachers with below average MKT and MTE. Each teacher's mathematics lessons were videotaped on three occasions. Lesson artifacts, such as copies of tasks, were gathered from each teacher. The teachers also completed the Mathematical Knowledge for Teaching (MKT) assessment in Number and Operations (K-6) (Hill et al., 2004) and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs et al., 2000).

The main focus of analysis was a cross-case analysis to describe the similarities and differences between mathematics instructional practices of the two cases of teachers. Additionally, studying each pair of teachers as a case revealed patterns within that case. Analysis was focused on three components of implementation of mathematics instruction (NCTM, 2007): tasks, discourse, and mathematical learning environment for sense-making. First, tasks were analyzed based on their level of cognitive demand. A higher level involves connecting procedures to underlying mathematical concepts or completing complex, non-algorithmic tasks. For lower level tasks, students perform exercises they have memorized or procedural steps with no connections to underlying mathematical ideas (Stein et al., 2000). Second, for analyzing discourse, Hufferd-Ackles, Fuson, and Sherin (2004) provide a framework for "levels" of math talk based on whether the teacher or the students are questioning, explaining mathematical thinking, the source of mathematical ideas, and responsible for learning. Third, to assess the mathematical learning environment of each classroom, instructional practices that appeared to contribute or deter from students' mathematical sense-making were noted.

Case #1: Profile One

The first case (Profile One) is presented through descriptions of Kristin and Andrea. Kristin, a Caucasian female, is in her seventh year of teaching. She has 25 students in her class (13 Caucasian, four Asian-American, three Hispanic-American, one Middle Eastern, one multi-racial). Andrea, also a Caucasian female, is in her tenth year of teaching. Andrea has 20 students (11 Hispanic-American, four African-American, three Middle Eastern, two Asian-American). They both teach all core subjects (Mathematics, Science, Language Arts, Social Studies) to their students. Compared to the mean of 103 third-grade teachers on the MKT and MTEBI assessments, Kristin and Andrea scored higher, indicating their MKT and MTE are stronger than many teachers in the larger sample. Based upon patterns noted in their instruction, three selected lessons are described to provide a sense of their mathematics instruction.

In one lesson, Kristin asked her students to make polygons from four congruent, right, isosceles triangles with the criterion that adjacent sides share a full side. Students worked in pairs to manipulate the triangles to form a variety of polygons, trying to make all fourteen possible shapes. In a whole-class discussion, Kristin led the class as they classified the shapes and examined common characteristics. As students brought shapes to the front of the class, Kristin placed each shape under one of four words: triangle, quadrilateral, pentagon, or hexagon.

Kristin categorized the first four shapes, but then, she asked the students to classify the remainder of the shapes. She asked her students to "turn to a partner" to discuss the common characteristics among the sorted polygons. The lesson concluded with a discussion about the common characteristics for each group of polygons.

In another lesson, Kristin asked the students to estimate the number of raisins in the box without looking inside. Then, they estimated how many raisins were in the box after opening it to examine the top layer, not removing any raisins. The class discussed their new estimates:

> *Mitchell*: I counted the layer on top and added the number of layers.
> *Kristin*: Cara, why did your estimate change?
> *Cara*: My estimate went up because the raisins were smaller than I thought.
> *Kristin*: Corinne, how did you change your estimate? What was your strategy?
> *Corinne*: I counted the layer on top with 10 raisins and counted nine layers. So, I did 10x9 which is 90.

Then, the students counted the number of raisins in the box by grouping them in some way to make them easier to count. Kristin asked the students to group them, circle a group, and write the number inside the circle in order to keep a written record. For the last 14 minutes, the class discussed how they grouped them and Kristin modeled how to write a division sentence. Kristin made an error when she wrote the equation $110 \div 11 = 10$ to represent a student dividing his 110 raisins into groups of 10 to obtain 11 groups. While her equation does not give an incorrect value, it does not match the student's thinking of $110 \div 10 = 11$. She made this same error five times for students' models.

In one of her lessons, Andrea focused on addition and subtraction story problems. For the first twenty minutes, the students solved an addition problem as a class and discussed the steps.

> *Displayed on board: 5,126 girls ran in the Reindeer Romp Saturday. 2,737 adults ran, too. About how many people ran in the race?*
> *Andrea:* Does this story want us to combine *(pulls her hands together)* or does it want us to take apart and find the difference *(pulls her hands apart and makes a "balance" motion with both hands)*?
> *Student*: Combine because it says how many people.
> *Andrea*: Yes, I think the how many people part *(underlines "how many people")*. It's the people part that we want to know, how many people, which means we take the girls and the adults and put them together.

Andrea focused her class on key words in the story problems. They determined whether the problem was asking them to combine the numbers or find the difference and whether the problem was asking for an exact or estimated answer. Andrea used gestures to represent addition, subtraction, exact, and estimate. For example, when deciding whether the problem

asked for an exact or estimated answer, Andrea gestured by smacking her hand on her leg when she said "exact", and she rotated her hand back and forth to represent getting an estimate. The students also made these gestures during the whole class discussion, and some students used the gestures when they worked in pairs during the next 50 minutes of class. The pairs solved two story problems and wrote two of their own. For the last 15 minutes of class, they sat in a circle and shared the stories they created.

Case #2: Profile Two

The second case (Profile Two) is presented through descriptions of Nancy and Catherine. Nancy and Catherine, both Caucasian females, are in their third and tenth years of teaching, respectively. They both teach all core subjects to their students. Nancy has 19 students in her class (9 Hispanic-American, five Asian-American, four Caucasian, one Middle Eastern). Catherine has 18 students (five multi-racial, four Caucasian, four Asian-American, three African-American, two Hispanic-American). Nancy and Catherine scored lower than the larger sample mean on the MKT assessment and MTEBI. Descriptions of three lessons for Profile Two follow.

One of Nancy's lessons started with students completing and reviewing a warm-up of six multiplication facts. For the next 13 minutes, they reviewed the answers to their homework on data and patterns to review for the state mathematics assessment. The class completed a worksheet on ordering digital clock times for about 10 minutes. Five of these 10 minutes were spent on non-mathematical conversation such as:

> *Nancy*: Who has been to Yellowstone? Does anyone know where it is?
> *Student #2*: Pennsylvania
> *Nancy*: It's not in Pennsylvania. It's in Wyoming. I've been to Yellowstone. There are big waterfalls. There are big buffalo.

During the next fourteen minutes, the students completed a worksheet called "Math Squares" on multiplication facts and reviewed their answers. The class concluded with a game called "Squish" on multiplication facts, which they played for five minutes.

In another lesson, the focus was equivalent fractions. For the first eight minutes, students wrote fractions for statements such as "Three of ten marbles are blue" as their warm-up. Students finished quickly so they read books during this time period while they waited. Next, Nancy spent ten minutes reading the book, *Full House: An Invitation to Fractions.* Then, she briefly reviewed the word "equal" and talked about "equal fractions". Next, the class spent sixteen minutes, under Nancy's direction, making paper fraction strips. They drew lines (through eyeballing) to divide

the strips into parts. Nancy drew fraction strip models on the board. The inaccuracies in her drawings included: her strips were not equal in length; her strips were divided into unequal parts since she sketched them quickly; and her strips did not show equivalent fractions (e.g., the $\frac{1}{3}$ on the thirds strip did not line up with the $\frac{2}{6}$ on the sixths strip). She posed a few questions to her students, requiring them to use the strips. One question was: "If you have pizza bread, would you want $\frac{1}{3}$ of the bread, $\frac{2}{6}$ of the bread, or $\frac{1}{4}$ of the bread?" A student responded that he would want $\frac{2}{6}$ of the bread because "2 is more than 1", referring to the numerators of the fractions. Nancy said and pointed to her inaccurate representations on the chalkboard, "Wouldn't you want any of them? Aren't they all the same size? They are. They're all the same size." Students spent the last 18 minutes on two worksheets on equivalent fractions.

A lesson in Catherine's classroom was focused on representing decimal numbers greater than one. After 10 minutes of calendar time, Catherine used 17 minutes to ask individual students to model a given decimal with base 10 blocks in the front of the classroom. As evident from another videotaped lesson, Catherine had used base 10 blocks earlier in the school year to represent whole numbers. The change to decimals requires assigning the blocks new values, which did not take place. For example, when a student accurately modeled 7.29 with seven flats, two rods, and nine unit blocks, Catherine said, "seven hundreds, two tens, and nine ones, 729 if it didn't have a decimal. So, it's seven point twenty-nine." Correct language would have been 'seven ones, two tenths, and nine hundredths which is equal to seven and twenty-nine hundredths'. Her descriptions of the blocks were not accurate, and she misnamed the place value of digits. She repeated this mistake for ten examples. Next, they reviewed problems in their textbook before spending the final 32 minutes on ten questions independently. Most students finished within 15 to 20 minutes.

Findings and Discussion

Our findings are grouped into similarities and differences between Profile One and Profile Two lessons. The differences between the two cases can be described in three overarching ideas: tasks, representations, and coherence. The similarities between the two cases fall under two ideas: discourse and demonstrated knowledge.

In terms of differences between the two profiles, first, tasks tended to be different. The teachers in Profile One tended to use conceptually-based tasks while Profile Two teachers' tasks

were more procedural in nature without connections to underlying mathematical ideas. For example, when Kristin asked her students to make polygons with four triangles, the level of cognitive demand was high as students applied their spatial reasoning skills and made connections about the composition of shapes. In contrast, Nancy asked her students to perform memorization exercises (multiplication facts), and Catherine engaged her students in non-conceptual procedures like writing decimals. Second, the Profile One teachers' use of representations tended to be appropriate and likely contributed to mathematical sense-making, unlike Profile Two. Kristin used a variety of representations for each type of polygon, and Andrea's gestures in the story problem lesson seemed to be particularly appropriate for her class with a relatively high percentage of English Language Learners. Nancy and Catherine used manipulatives (i.e., fraction strips, base 10 blocks) inappropriately due to their mathematical inaccuracies, which likely hindered their students' mathematical sense-making. Third, lessons of Profile One teachers were much more coherent pedagogically and mathematically which likely impacts students' sense-making and hence, opportunities to learn. Kristin and Andrea tended to maximize time on-task, focus on one mathematical concept in a lesson, and deliver mathematically accurate instruction. In contrast, Nancy and Catherine had non-mathematical conversations (e.g., Yellowstone), their activities did not build on each other, and they made glaring mathematical mistakes (e.g., saying nonequivalent fractions were equivalent, assigning incorrect place value).

For similarities, first, teachers in Profiles One and Two facilitated similar discourse. The "Levels of the Math-Talk Learning Community Framework" (Hufferd-Ackles et al., 2004) describe a Level 0 classroom as a traditional teacher-directed classroom with short responses from students. A Level 1 classroom means the teacher pursues some mathematical thinking, but he or she still plays a large role in the mathematical talk. Kristin's and Andrea's classrooms would fit between Levels 0 and 1. Much of their whole-group discussion fits the Level 0 descriptors, but they offered students some time to talk to each other about mathematics (e.g. "turn to a partner") and pushed for some thinking. Nancy's and Catherine's classrooms fit the Level 0 descriptors because their questions required short responses with no explanation of mathematical thinking. Second, both Profile One and Two teachers promoted misconceptions among students. For example, Andrea used the key word strategy that we know is often misleading (Van de Walle et al., 2010). Nancy's inaccuracies in her fraction strip drawings on

the board were very misleading for students in terms of what they conceived about fraction equivalencies. The promotion of mathematical misconceptions seems to impact students' opportunities to learn mathematics.

The work of this study reaffirms previous research about the importance of the teacher in mathematics instruction. The tasks, representations, coherence, discourse, and the teacher's demonstrated knowledge play a role in the students' opportunities to learn mathematics. While we cannot make causal statements, it appears teachers with higher levels of MKT and MTE are more likely to use cognitively demanding tasks, appropriate representations, and a coherent set of lesson activities with no mathematical inaccuracies. It is likely that teachers with higher MKT feel more efficacious in their teaching. It appears the facilitation of high-quality discourse and avoiding the promotion of misconceptions may be challenges, regardless of MKT and MTE. These findings imply three foci for mathematics educators and professional development (PD) providers. First, teacher education and PD programs need to continue the work to deepen MKT among teachers. It is likely that stronger MKT will improve MTE simultaneously. Second, a focus on discourse in teacher education or in PD with opportunities to observe, practice, and reflect on high-quality discourse appears important, even for teachers with deeper MKT. Finally, attention to teachers' knowledge of the vertical curriculum seems appropriate in order to decrease the promotion of misconceptions.

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OVERCOMING A COMMON STORM: DESIGNING THE PD TEACHERS NEED FOR SUCCESSFUL COMMON CORE IMPLEMENTATION

Gabriel Matney
Bowling Green State University
gmatney@bgsu.edu

Jonathan Bostic
Bowling Green State University
bostic@bgsu.edu

Daniel Brahier
Bowling Green State University
brahier@bgsu.edu

Classroom implementation of the Common Core State Standards (CCSS) requires significant professional development that is sustained over time, develops teachers understanding of the Standards for Mathematical Practice, and begins with the content and professional needs of the teachers it serves. This study examines elementary and middle school teachers' perceived content needs related to the CCSS mathematics content domains, their perceived professional needs, and the connection between these perceptions and statewide assessment data. K-5 teachers indicated a great need in Operations and Algebraic Thinking and Numbers and Operations on Fractions. Middle school teachers expressed a major need in better understanding modeling, statistics and probability, geometry and measurement, and proportional reasoning. K-9 teachers perceived professional needs and implications for designing professional development for inservice teachers are discussed.

The recent adoption of the Common Core State Standards (CCSS) by 45 of the 50 states will lead to major instructional changes in K-9 classrooms. Proper implementation of the CCSS demands much more than revised textbooks. Standards of Mathematics Content and Mathematical Practice are different from prior state standards (Chief Council of State School Officers, 2010), hence instructional materials and practices must adapt to these new expectations. Sustained professional development (PD) for teachers will help them acquire the mathematical knowledge necessary to fully instantiate the intent of the CCSS to facilitate these changes (Wu, 2011). The purpose of this paper is to examine teachers' perceptions of needed PD as they move toward implementing the CCSS.

Related Literature

Teachers are the critical instructional element in the classroom (National Council of Teachers of Mathematics [NCTM], 2000). They manage instructional norms, discourse, tasks, and tools (Franke, Kazemi, & Battey, 2007). They are also expected to deeply understand mathematics, mathematics pedagogy, and potential outcomes for students (Mewborn, 2003). PD aims to support teachers to maintain effective instructional contexts and adapt to new challenges. Sustained PD like QUASAR (Stein, Silver, & Smith, 1998) that goes on for over a period of months and gives teachers a safe, supportive environment to explore pedagogical and content issues has led to meaningful student and teacher outcomes (Mewborn, 2003). Sustained PD that aims to support K-12 mathematics teachers' pedagogical content knowledge is likely to enhance

students' outcomes and leads to long lasting teacher change (Ball & Bass, 2000). Results from large scale survey research with teachers indicated that PD focusing on (a) content knowledge, (b) opportunities for active inquiry-based learning, and (c) coherence within this PD leads to positive changes in teachers' classroom practices (Garet, Porter, Desimone, Birman, Yoon, 2001). In light of this evidence, teachers need support to refine and improve their instructional practices to implement the recently adopted CCSS.

The CCSS emphasize student reasoning and understanding of mathematics throughout K-9 instruction (CCSSO, 2010). NCTM has advocated for reasoning and sense making throughout K-9 mathematics instruction as well as effectively assessing students' mathematical understanding (NCTM, 2010; 2009; 2007; 2006; 2000). Knowing that teacher educators have these and other resources from which to design rich professional developments to enhance teachers' pedagogical content knowledge (Shulman, 1986), we intended to design PD for teachers focusing on the CCSS and were interested to use teachers' perceived needs as a key rationale for its structure and content. Toward this aim the Standards for Mathematical Practice are seen as a vital element that must be included, collectively, within any PD that is designed to help teachers implement the CCSS. Teaching any of the Standards for Mathematical Practice separately from the context of content is likely to not have lasting effects much like the heuristic instruction movement (Lesh & Zawojewski, 2007). These practices for mathematics learning were derived from NCTM's (2000) five process standards and the National Research Council's (2001) five strands of mathematical proficiency and provide the important lens through which the teaching and learning of particular mathematics content should be viewed. Therefore, the Standards of Mathematical Practice should be the unifying thread that runs throughout PD as teacher educators deepen and enrich practitioners' content knowledge on particular common core topics.

The Standards for Mathematical Practice will be the inherent focus in each piece of the PD, yet there still remain important delineations to consider before design. The full breadth of content knowledge in either the elementary or middle grades CCSS would require vastly more time than most PDs can offer. Furthermore teachers may want help in particular professional areas as they relate to the CCSS. Again, the feasibility of teacher educators to incorporate the many needs is strained by the typical duration and scope of PD. Noting these time constraints we sought to better understand teachers' perceived needs during this transition to CCSS in order

to design meaningful, coherent, and relevant PD for the teachers we serve. Our research questions are (1) Which of the K-9 content standards do teachers perceive the greatest need for professional development? and (2) What specific professional content features do teachers perceive they need the most from PD?

Method

Participants

The participant population was K-9 teachers of mathematics spread across four different counties of a state in the Midwest Region of the United States. The participant population was further stratified K-5 (Elementary Cohort) and 6-9 (Middle Cohort) in order to better group the CCSS mathematical domains. The four counties exhibit a wide range of population types including urban with low median income/high poverty and rural/agricultural with high poverty.

The Middle Cohort in this study included ninth-grade teachers due to statewide licensure factors of the state in which the research was conducted.

In the Elementary Cohort all 469 teachers were asked to voluntarily participate in the survey by their administrators. Nearly one third of that cohort responded and answered the survey (n=148). There are twenty-two grades 6-9 mathematics teachers in the Middle Cohort. All of them volunteered to complete the survey. The number of teachers participating in the survey at each grade level can be found in Table 1.

Table 1.
Number of Participants by Grade Level

Grade	n
K	25
1	31
2	30
3	21
4	22
5	19
6	6
7	2
8	1
9	13

Throughout the year prior to the study each of our partnering districts provided information to the teachers about the CCSS and the degree to which they aligned with current state standards in use. Teachers were familiar with the common core domains, clusters, and standards.

Instrumentation

Two different surveys were created to ascertain perceived needs from teachers in Elementary and Middle Cohorts. This was done after examining the CCSS mathematics domains and determining that standards were fairly consistent for K-5 and 6-8 grade bands.

The data collected for this study focused on teachers' perspectives on mathematics content and professional needs via an anonymous survey. Survey items asked participants about their district, grade levels taught, and years of teaching experience. Participants also rank ordered the K-9 CCSS mathematics domains and desired professional development focus. Finally, they indicated their level of interest in participating in sustained professional development about these topics.

The participants in the Elementary Cohort were asked:

1) In which school area do you teach?
2) How many years of mathematics teaching experience do you have?
3) What grade level(s) are you currently teaching?
4) Rank the following K-5 Common Core Mathematics Content Standard areas IN ORDER, where "1" is the Standard you feel that you need the MOST and "6" means you need the LEAST help in implementing that standard: Counting and Cardinality, Operations and Algebraic Thinking Operations, Numbers and Operations in Base 10, Numbers and Operations - Fractions, Measurement and Data, Geometry.
5) Rank the following 7 areas of mathematics professional development IN ORDER, where "1" is the topic of MOST interest/value to you and "7" means you currently have the LEAST need for help in that area: Enhancing or deepening my understanding of the Common Core, Helping students to reason and make sense of mathematics, Use of technology in teaching mathematics, Improving instructional strategies for student conceptual development, Collaboration with other mathematics teachers, Web Sites useful for planning and teaching mathematics, Diagnostically assess students' understanding in order to plan lessons or interventions.
6) A grant is being written to provide professional development for teachers of mathematics throughout 2012. How likely would you be to participate: Definitely Interested – count me in, Greatly Interested – depends on some factors but very likely, Somewhat Interested – I would need to think about it, Probably Not – I'm not sure I have the time or interest to participate at this time, No – count me out (Matney, 2011).

The survey questions for the Middle Cohort were similar except for questions four and five. This choice was due to different levels of instructional content and different potential

professional needs. We worked with the school districts to include items the curriculum specialists, who have professional contact with the participants, thought would be of interest to the teachers at differing levels of elementary and middle school. The modified content and professional needs for the Middle Cohort were:

1) Ratios and Proportional Reasoning, Geometry, Statistics and Probability, Number System/Number and Quantity, Algebra, Functions, Modeling.
2) Using technology in mathematics, The Common Core State Standards, Supporting students to reason and make sense of mathematics, collaborating with other mathematics teachers, Instructional Strategies (Bostic, 2011).

Data Collection

The surveys were sent to district administrators (e.g., superintendents, curriculum coordinators, and principals) to disseminate to mathematics faculty in their district. Teachers were encouraged to complete the survey during a two-week window.

District-level data were also collected to examine the degree to which teachers' perceived needs matched students' performance on statewide mathematics assessment from the prior academic year. Students' statewide assessment performance is collected from third through eighth grade. The mathematical subgroupings found on the statewide mathematics assessment closely align with the CCSS mathematics domains.

Data Analysis

The following approach was used to determine an overall score for the two questions focusing on mathematics content and professional needs based on the percentage of participants selecting that rank. First, the ratio of responses to total responses was calculated for each content and pedagogical domain and each rank order. This ratio was multiplied by 100 to determine the percentage of participants indicating that response. Next, the percentage was multiplied by its rank order (e.g., six for definite need, five for great need, four for some need, …, one for no need) and these values for a particular content or professional needs domain were summed to determine an overall score.

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Results

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Perceived Needs of the K-5 Elementary Cohort

    Content

The K-5 group of teachers rank ordered the following CCSS mathematical domains from greatest need to least need and percentages for each response are presented in Table A1 of

Appendix A. Teachers indicated that the two most important areas for content development were Operations & Algebraic Thinking and Numbers & Operations on Fractions. The domain of Measurement and Data was a close third choice. These perceived needs align with students' statewide assessment performance in that they represent content choices in which the students of these teachers have been shown to struggle via statewide assessments. Approximately 18% of third-grade students failed to meet the state required proficiency. However, the fourth- and fifth-grade failure rates were much higher; 24% and 42, respectively. When the level of mathematical sophistication increases on the state assessment, in the areas of algebra and fractions, the students' failure rate on the overall exam also increases.

Professional Needs

The overall professional needs score (see Table A2 of Appendix A) gives a strong sense that teachers desire (a) a better understanding of the CCSS, (b) ways to encourage students' reasoning and sense making, and (c) improving their instructional strategies to facilitate conceptual development. Teachers perceived their need for better understanding of the CCSS as the highest. The next two highest choices of *student reasoning* and *conceptual development* support teachers' first choice since they are closely associated with the CCSS and are pertinent to understanding its implementation through the Standards for Mathematical Practice. Finally, 58% of the teachers surveyed indicated that they were "definitely" or "greatly" interested in long term professional development over these perceived needs.

Perceived Needs of the 6-9 Middle Cohort

Content

Teachers overwhelmingly asked for PD focusing on modeling, which is woven throughout the CCSS (see Table A3 in Appendix A). Statistics and probability, geometry and measurement, and proportional reasoning were also perceived as areas of great need. The statewide assessment results from the previous year indicate that approximately 15% - 29% of grades 6-8 students were not proficient on data-related tasks and 14% - 46% of grade 6-8 students did not meet passing criteria on geometry and measurement tasks. Modeling tasks were embedded throughout the assessments as word problems that require making sense of text, creating suitable models, and solving the task. Thus, no data were available from statewide assessments indicating students' modeling or problem-solving performance.

Pedagogy

Middle school teachers clearly valued PD focused on some professional topics more than others, as shown in table A4 of Appendix A. Teachers were most interested in learning about ways to help students reason and make sense of mathematics. PD focused on instructional strategies to promote students' conceptual development and enhancing their knowledge about the CCSS was also perceived as valuable. Finally, 59% of participants stated they would "definitely" participate in sustained PD.

Discussion and Implications

K-5 and 6-9 teachers indicated different content-specific needs. K-5 teachers perceive needing PD focused on topics typically taught during later elementary years, such as algebraic thinking and operations with fractions. Middle school teachers expressed clear need for a better understanding of modeling. Modeling impacts one's understanding and ability to solve word problems, which is embedded throughout nearly every content strand. The CCSS for Mathematics Content frequently reference applying one's knowledge to solve real-world problems, which requires modeling. Finally, participants tended to respond in ways that were similar to their students' outcomes on statewide tests.

Statewide assessments involve progressively more sophisticated mathematics content as grade levels increase. For the Elementary Cohort the two lowest content needs were Counting and Cardinality and Numbers and Operations in Base 10 which are in large part completed by third grade. Therefore it is noteworthy that students performed the best on the third grade state assessment with 82% meeting state proficiency while fourth and fifth grade state proficiency rates were 76% and 58% respectively. This indicates that students' ability to demonstrate proficiency with lower elementary grade ideas matches the teachers ranking these as low priorities.

On the other hand, the highly requested content topics are deeply developed during the latter elementary and middle grades. These topics are also given richer treatment on the statewide assessment in grades 4 and 5. Only 14.9% and 12.8% of the participants surveyed were fourth and fifth grade teachers. The vast majority of the teachers (i.e. 72.3%), in the Elementary Cohort taught primary elementary grades yet still recognized the need for PD focusing on preparing students for intermediate elementary content. Thus, elementary teachers' perceived needs for PD

about CCSS mathematics content domains align with their students' prior performance on statewide assessments.

There was a noticeable increase in the number of students not meeting proficiency on high stakes tests from sixth- to seventh- and eighth grade. The districts' average sixth-grade below-proficiency score was 21% whereas 35% and 34% of seventh- and eighth-grade students on average did not meet proficiency on their mathematics tests. A cursory inspection of the data also suggests some tentative association between students' proficiency scores and the content areas teachers requested. The average below-proficiency score related to geometry and measurement increased as grade levels increased from grades six through eight (i.e., content is more complex): 19%, 31%, and 32% respectively. Data and analysis below-proficiency average scores were more consistent across sixth-, seventh- and eighth-grade: 21%, 24%, and 24%. Curriculum coordinators remarked that modeling was woven throughout the high-stakes tests in the form of word problems that drew on a variety of content areas. For example, one coordinator reported that data analysis tasks typically require students to read a problem's stem, interpret a table and graph, and make judgments about appropriate procedures and conclusions. Thus, middle grades teachers' expressed desire for PD focusing on instruction that supports students' problem solving and reasoning and sense making within the context of these content areas seems aligned.

K-9 teachers have similar perceived professional needs for PD. That is, both cohorts want PD focused on understanding the CCSS, helping students to reason and make sense of mathematics, and to explore instructional strategies focused on students' conceptual development. These needs align with the CCSS, which indicate that positive problem-solving behaviors are necessary to learn mathematics deeply. The adoption of new standards also provides teacher educators an opportunity to support instructors teaching to the new standards, and there is a fervent perceived need for PD focusing on these topics.

Teacher educators developing CCSS-focused PD should consider teachers' perceived needs. Teachers and curriculum coordinators should also be a part of the PD planning process. There is clearly a demand from teachers to learn more about ways to support students' reasoning and sense making, which includes teaching strategies that support student-centered, inquiry-focused instruction. As a result of this work, we crafted a grant funded PD program for K-9 teachers and will implement PD focusing on teachers' perceived needs.

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Appendix A

Table A1
Perceived Mathematics Content Needs of the Elementary Cohort

CCSSM Domain	Definite (%)	Great (%)	Some (%)	Fair (%)	Little (%)	No (%)	Overall Score Max = 600
Operations and Algebraic Thinking	37.16	20.27	10.81	18.24	10.81	2.70	446.62
Numbers and Operations – Fractions	14.19	26.35	27.03	13.51	11.49	7.43	395.95
Measurement and Data	17.57	18.24	20.95	23.65	12.16	7.43	383.11
Geometry	10.14	18.92	12.16	26.35	20.95	11.49	336.49
Numbers and Operations in Base 10	6.08	12.84	22.97	12.16	41.89	4.05	316.89
Counting and Cardinality	14.86	3.38	6.08	6.08	2.70	66.89	220.95

N = 148

Table A2
Perceived Professional Needs of the Elementary Cohort

Professional Need	Definite (%)	High (%)	Great (%)	Some (%)	Fair (%)	Little (%)	No (%)	Overall Score Max = 700
Teaching CCSS	38.51	9.46	9.46	8.78	10.81	8.11	14.86	472.30
Supporting reasoning and sense making	24.32	16.89	12.84	18.92	9.46	10.81	6.76	468.24
Using technology	10.81	18.92	16.22	12.84	16.22	11.49	13.51	406.76
Instructional strategies	11.49	21.62	29.73	12.16	13.51	9.46	2.03	468.92
Collaborating	1.35	8.11	10.81	14.86	11.49	25.68	27.70	285.14
Web Support	5.41	8.78	12.84	18.92	14.86	20.27	18.92	334.46
Diagnostic Assessment	8.11	16.22	8.11	13.51	23.65	14.19	16.22	364.19

N = 148

Table A3
Perceived Mathematics Content Needs of the Middle School Cohort

CCSSM Domain	Definite (%)	High (%)	Great (%)	Some (%)	Fair (%)	Little (%)	No (%)	Overall Score Max = 700
Modeling	22.22	16.67	16.67	16.67	22.22	5.56	0.00	483.33
Statistics and Probability	11.11	11.11	16.67	27.78	22.22	11.11	0.00	427.78
Geometry	16.67	11.11	11.11	27.78	16.67	5.56	11.11	422.22
Proportional Reasoning	7.14	14.29	14.29	21.43	35.71	7.14	0.00	414.29
Algebra	10.53	0.00	31.58	0.00	21.05	26.32	10.53	357.89
Functions	11.11	0.00	5.56	33.33	5.56	27.78	16.67	327.78
Number and Quantity	6.25	0.00	12.50	18.75	6.25	37.50	18.75	293.75

N = 22

Table A4
Perceived Professional Needs of the Middle School Cohort

Professional Need	Definite (%)	Great (%)	Some (%)	Fair (%)	Little (%)	No (%)	Overall Score Max = 600
Supporting Reasoning and Sense Making	44.44	44.44	11.11	0	0	0	533.33
Using technology	55.56	22.22	16.67	0	5.56	0	522.22
Teaching CCSS	44.44	27.78	11.11	16.67	0	0	500
Collaborating	27.78	16.67	44.44	5.56	5.56	0	455.56
Instructional strategies	11.11	27.78	33.33	16.67	0	11.11	400

N=22

REASONING, SENSE MAKING, AND PROFESSIONAL DEVELOPMENT

Daniel J. Brahier
Bowling Green State University
brahier@bgsu.edu

Debra Gallagher
Ohio Northern University
d-gallagher.2@onu.edu

A professional development program for approximately 30 teachers in the Midwest U.S. was conducted to promote implementation of the Common Core State Standards for Mathematics. After nearly 100 hours of professional development, teachers in the project demonstrated a significant positive change in their attitudes toward the teaching of mathematics. Other data on student achievement and attitudinal changes have been collected and are still being analyzed.

In 2006, the National Council of Teachers of Mathematics (NCTM) released a pivotal document titled *Curriculum Focal Points* that delineated key content benchmarks for each of the grades from kindergarten through eight. This book became the topic of much debate, as educators attempted to come to some level of consensus on the expectations of all students at each grade level. Furthermore, the publication of *Focal Points* inevitably resulted in discussion about the need for similar expectations for all high school students. In response, *Focus in High School Mathematics: Reasoning and Sense Making* was released three years later (NCTM, 2009). Instead of describing specific content-related outcomes, the more recent publication described the type of mathematical thinking that should be emphasized in the high school classroom. Using a series of content-specific examples, the authors painted a picture of the ideal secondary classroom in which students are challenged to develop reasoning skills that can be used in college and the workplace.

By the summer of 2010, the National Governors Association Center for Best Practices and the Council of Chief State School Officers released the *Common Core State Standards for Mathematics*, adopted by more than 45 states as their core mathematics curriculum (Common Core State Standards Initiative [CCSSI], 2010). The Common Core features a series of standards, organized into clusters by domain areas and listed by grade level (conceptual categories at the high school level). In addition, the document described eight mathematical practices that should be embedded into the teaching of all mathematics, such as "make sense of problems and persevere in solving them" and "reason abstractly and quantitatively" (CCSSI, 2010, p. 6).

Recognizing the difficulty that teachers in local school districts – particularly at the upper grades – would have in rethinking mathematical content, sequence of topics, and approaches to promoting reasoning and sense making, a professional development program was proposed for

teachers in two counties in the Midwest U.S. With approved funding through the Improving Teacher Quality program, the project entitled Common Core for Reasoning and Sense Making – $(CO)^2$ RES – was launched in March of 2011. The goals for the project included the following: Teachers will become familiar with the format and content of the Common Core and new State-level Standards and demonstrate teaching strategies that promote reasoning and sense making in the mathematics classroom. In addition, students will demonstrate an increase in achievement on free-response and short-answer questions in mathematics that require problem solving and reasoning skills and develop productive and positive dispositions toward mathematics.

A total of 31 teachers remained in the program throughout the year, participating from districts representing a mix of urban, suburban, and rural settings. Teachers in the project taught mathematics in the range of grades 6 through 12. Professional development workshops were presented in the spring semester; a two-week summer institute was run in the summer, and a fall semester follow-up series of workshops and electronic exchanges were conducted. By the conclusion of the program, teachers had more than 100 contact hours of professional development. These experiences consisted of presentations by experts in pedagogy and content knowledge, hands-on sessions with manipulatives, handheld technology, and computer software, and designing of inquiry-based lessons that were developed to promote reasoning and sense making and address standards in the Common Core State Standards.

Research questions were identified to study the effects of the $(CO)^2$ RES project on the teacher participants as well as the students in their classes, as follows:

1. What attitudinal effects toward the nature of mathematics and the use of inquiry did the $(CO)^2$ RES project have on the teacher participants?
2. What attitudinal effects toward the nature of mathematics did the teacher participants have on their middle and high school students?
3. To what degree were the students of participants affected in their ability to respond to high-level mathematical problems that required reasoning and sense making?

Related Literature

Student achievement is the goal all educators are seeking. Much research has been done over the past several decades to determine the factors that most significantly affect student achievement. Class size, school size, and socioeconomic level all play a part in student achievement (Darling-Hammond, 2000), but quality classroom instruction is the single most

important contributor to achievement (Odden & Wallace, 2003). An effective teacher has six to ten times as much impact on achievement than other factors combined (Mortimore & Sammons, 1987). Many elements, such as intelligence, verbal ability, subject matter knowledge, knowledge of teaching and learning have all presumed to be indicative of teacher quality (Darling-Hammond, 2000), but that is beyond the realm of this study.

In addition, the quality of professional development (PD) of teachers may positively influence student achievement. While research shows that not all PD programs, even after two years, have a statistically significant impact on teacher knowledge or on student achievement (Institute of Educational Sciences, 2011), participation in sustained professional development influences teaching and, ultimately, student achievement (Darling-Hammond, 2000). Reeves (2010) outlines professional learning in three essential areas: (1) Focus on student learning, (2) Rigorous measurement of adult decisions, and (3) Focus on people and practices, not programs (p. 21). Professional courses need to be learner centered, allow opportunities to try out new things, and be reflective.

Another feature contributing to students' understanding of mathematics is the teacher's attitude toward teaching the subject. Yara (2009) states, "Teachers' attitude towards the teaching of Mathematics plays a significant role in shaping the attitude of students towards the learning of Mathematics" (p. 365). An individual's attitude toward teaching mathematics may be determined by the perceived ability the teacher has to implement change in a specific task or situation (Bandura, 1997). Teachers with lower levels of mathematical content knowledge may have negative attitudes toward teaching mathematics, thus increased content knowledge can be a way to positively influence teacher attitudes. One study (Swackhamer, et al, 2009) showed significantly higher Teaching Outcome Expectancy for teachers who took had taken four or more mathematic courses than those who took only one to three courses.

Ultimately, the goal of the $(CO)^2$ RES project was to actively engage teachers in doing mathematics and then reflecting on the content and pedagogy used in their own experiences. As teachers experienced the power of physical and visual models, technology, and sense making, they were expected to develop a vision for their own classrooms on how to best implement the Common Core State Standards.

Methodology

Data were collected to address the research questions in a variety of ways. On the first day of the project (early March), teachers completed a survey that asked questions addressing their beliefs about the nature of mathematics and teaching. The instrument used to measure attitudes and beliefs about mathematics teaching and learning was the *Mathematics Attitude Survey,* a 33-question survey using a 5-point Likert Scale (1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly Disagree). Reverse scoring was used for 17 negatively worded items. This instrument was an adaption of an instrument developed for the Maryland Collaborative for Teacher Preparation by Dr. Randy McGinnis (1998). The following are examples of positively worded items: (4) I like doing mathematics and (13) I know I understand mathematics when I can explain the mathematics to someone else. Examples of negatively worded items include: (10) People learn mathematics by listening to lecture and (25) Being able to successfully use a rule or formula in mathematics is more important than understanding why the rule of formula works. In addition to the survey questions, demographic information such as age, number of years of teaching, and familiarity with the Common Core Standards for Mathematics was also collected.

At the end of the project – approximately 9 months later – teachers (N=29) completed the same survey. A t-test comparing pre- and post-testing indicated a statistically significant change in attitudes about mathematics teaching over the course of the program. The mean pre-test score was 3.8472 and the mean post-test score was 4.0250, $p<.05$. Participants commented on strengths and weaknesses of the program, as well as what they perceived to be challenges of teaching mathematics and how these difficulties were addressed through the project. Triangulation of the qualitative (focus groups) and quantitative (survey results) offers a more complete picture of the impact of the program on the teacher participants.

In addition, the effects of the program on the students in grades 6 through 12 who were taught by project participants were explored. In the first week of the school year (following the summer institute), students were administered an attitudinal pre-survey. The *Student Attitudes About Mathematics Instruction* pre-survey is a 17-question survey using a 5–point Likert Scale similar to the teacher survey. Reverse scoring was used for 7 of the negatively worded items. Additional questions such as "What grades do you usually get in mathematics" and "How much effort do you usually put into your mathematics work?" were asked. Negatively worded questions included: (d.) Doing mathematics often makes me feel nervous or upset and (k.)

Knowing mathematics doesn't help get a job. Positively worded questions included: (a.) I enjoy mathematics and (c.) Mathematics challenges me to use my mind.

Approximately three months later, the students completed a post-survey. The post-survey included additional items that provide information about the mathematics class the student in currently taking are scored on a 5-point Likert Scale (1=Never, 2= 1-2 times a month, 3=1-2 times a week, 4= almost every class, 5= every class) such as: (a.) Worked on hands-on activities, such as using manipulatives and (b.) Used calculators or computers for learning, practicing skills, or solving problems. Other items provide information about the teacher in this mathematics class scored on a 5 point Likert Scale (1=Strongly Disagree, 2=Disagree, 3=Not Sure, 4=Agree, 5=Strongly Disagree), including (a.) Really enjoys teaching mathematics and (b.) Makes mathematics exciting.

Students completed a five-item pre-test of free-response mathematical problems. These questions were modeled after free-response questions asked on the state's Graduation Test. Three levels of the free-response problems were created. Level 1 was given to students in grades 6 and 7; Level 2 was given to students in grades 8 and 9, and Level 3 was given to students in grades 10-12. An example of a Level 1 free-response question, graded on a 4-point rubric, follows:

A. Jillian sees that shampoo, hair gel, and toothpaste are on sale. The signs show the regular price and the discount.

SHAMPOO	HAIR GEL	TOOTHPASTE
Regular Price $7.50 1/3 off	Regular Price $4.50 1/2 off	Regular Price $6.00 25% off

Determine the amount that Jillian will save when she buys one bottle of shampoo, one container of hair gel, and one tube of toothpaste at the sale prices. Show or explain how much she will save.

At the conclusion of the project, students completed an attitudinal post-survey and a post-test of mathematical problems. Both attitudinal survey and test results are being compared to measure the impact of the teacher on the students during the three months of contact with teacher participants.

Findings

An analysis of the pre- and post- *Mathematics Attitude Survey* that the teachers took revealed a significant positive change in the attitudes of the teachers toward teaching mathematics. At this time, all pre-and posttest data, for the students are being analyzed. Final results will be reported at the annual meeting of the Research Council on Mathematics Learning and subsequent publications. However, formative program evaluation data were collected that shed some light on the impact of the $(CO)^2$ RES project. Specifically, the Project Evaluator observed PD sessions and conducted structured interviews with the 31 teacher participants during the summer institute. Participants were asked what was the most useful aspect of the program. Teachers gave a variety of responses, most of which focused on their deeper understanding of the Common Core State Standards and how to implement them. For example, one teacher stated:

> *After the Spring Semester, I put the textbook on the shelf and I said I know what math is supposed to be like and the textbook is not what this math is supposed to be like. It has kinda given me permission to teach math the way I (now) know it's supposed to be taught.*

In a sense, teachers felt a freedom to stray from the textbook and extend more deeply into problems in their classrooms. In addition, many participants spoke of the importance of forming community and working with their colleagues in the project. In the words of one participant:

> *When you go (to professional development) with mathematics teachers, (you get) the opportunity to share … (it's) that community that you are involved in at the different grade levels and across the levels.*

Furthermore, when teachers were asked about what they are using most in their classrooms, many spoke of the power of focusing on reasoning and sense making. For example, one participated stated, "If you're spending time doing reasoning and sense making it will carry over onto the next (unit)," recognizing that mathematical practices cut across all content areas. In turn, reflecting on what students are gaining from the changes in their teaching practice, most agreed that the changes were having a positive impact. In the words of one participant:

> *My students overwhelmingly let me know how much happier they were in class when we went to the bigger problems and put the textbook away. Math was more fun (for them), and they told me they're really looking forward to math.*

Finally, reflecting on the program as a whole, teachers were pleased with the opportunity to get "ahead of the curve" by learning about the new standards and how to promote reasoning and sense making ahead of others in their districts. Comments included:

A big part of the emphasis of the program was to learn about the new standards, and I never would have taken the time to sit and look through them if it wasn't for something like this.

In many cases, teacher participants stated that they did not realize how much work the program would be when they signed up but were glad to have been part of the process.

Discussion

The release of new national standards and NCTM documents necessitate professional development for teachers. If teachers are going to be expected to implement the standards and promote reasoning and sense making, they need to formulate a vision of mathematics teaching that becomes their goal. The $(CO)^2$ RES project was developed to assist the teachers in creating that vision. Preliminary results are mixed, as teachers continue to look for specific ways to implement the standards and mathematical practices in their own classrooms. They believe the program is worthwhile; many are already seeing changes in their own teaching and in the success levels of their students. They appreciate the opportunity to work with other teachers and to conduct cross-grade-level discussions within and among districts. However, many feel consumed by the level of work that is required to understand the standards and how to implement them.

It is important to note that the participants in this project voluntarily chose to register. One might expect that the "first wave" of teachers signing up for such a program tend to be those who are most interested in mathematics and changing their teaching practices. Given that the program only serves a fraction of the total teacher population in the target districts, the monumental task of bringing all teachers forward becomes much more evident. In other words, if the best teachers who are hungriest for professional development are feeling somewhat overwhelmed by the standards and the learning process, one can only speculate on how difficult it might be to bring all of the teachers in a district forward in the process. In the end, a comprehensive plan will need to be developed by each district to nurture change at all levels. It is the hope of the Project Director that the $(CO)^2$ RES teachers will serve as professional development providers and support teachers in this process. And, in fact, one county-level professional development program has already involved presentations by several of these teachers to assist others to formulate a vision for teaching mathematics.

What remains to be seen is the impact the project will have on student achievement in mathematics. Will the $(CO)^2$ RES teachers' students outperform their peers on standardized tests and problem solving tasks over the next couple of years as they implement the standards and practices? Additional research will be needed to provide experimental and control group data to validate the improvement of student success by teachers in the project. However, at the present time, the need for the PD is recognized, has been addressed, and appears to be having a positive influence on this group of teachers. Expanding the project to other teachers and districts, while collecting additional attitudinal and student achievement data, will help to steer the direction of such PD programs over the next several years as the Common Core Standards are phased in.

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STUDENT STRATEGIES SUGGESTING EMERGENCE OF MENTAL STRUCTURES SUPPORTING LOGICAL AND ABSTRACT THINKING: MULTIPLICATIVE REASONING

James A. Carrier
University of North Carolina at Greensboro
jacarrie@uncg.edu

Sarah B. Berenson
University of North Carolina at Greensboro
sarah_berenson@uncg.edu

Here we build upon the research of Clark and Kamii, (1996); Confrey, (1994); Harel and Sowder, (2005) and many others who investigate the development of multiplicative reasoning. This study examines the thinking of 14 fourth graders and concludes that the levels of multiplicative reasoning can be further defined in terms of indicators and refined in terms of student strategies.

The difficulties encountered by students in their transition to advanced mathematical thinking may be explained by a lack of understanding of many concepts taught in early school years, especially multiplicative reasoning (For example Confrey, 1994; Dreyfus, 1991; Harel & Sowder, 2005). By its nature, advanced mathematical thinking relies on a cumulative foundation of prior mathematical experiences. Students cannot comprehend advanced mathematical topics such as differential equations unless they understand underlying concepts, such as differentiation. Differentiation requires conceptual understanding of the idea of functions and assumes the student understands variables. Understanding variables is dependent upon the students' understanding of number, which is dependent upon understanding of quantification, which requires comprehension of serial correspondence. In other words, there is a structural order with each previous topic serving as a foundation for the next levels of mathematics and therefore we focus here on investigating children's multiplicative reasoning.

The concept of unit with respect to addition is quite different than the concept of unit with respect to multiplication. Addition and multiplication involve hidden assumptions. The hidden assumption in addition is that the unit is one, which children readily understand; whereas, the hidden assumption in multiplication is that the unit is one as well as more than one simultaneously (Chandler & Kamii, 2009). Splitting, Confrey's (1994) contribution to understanding multiplicative reasoning, speaks to conditions under which reunitizing occurs after the split in multiplication. Park and Nunes (2001) suggest that children's concept of multiplication originates in their schema of correspondences and not in the concept of addition. Under this definition the concept of multiplication is defined by a constant relationship between two quantities known as ratio and is a core meaning of multiplicative reasoning. The ratio or rate

is the constant unit that is called the multiplicand and acted upon by the multiplier. Children employ the schema of correspondence in order to represent fixed relationships between variables and solve multiplication problems.

Clark and Kamii (1996) identified five developmental levels that can be systematically observed in participants progressing from additive to multiplicative thinking. We propose an expansion of the multiplicative reasoning markers listed in Table 1 to a more detailed list of markers. Our focus is to identify observable behaviors of fourth grade students to expand the ideas found in Table 1. Towards this goal we ask the following questions:

1. What are the indicators of multiplicative reasoning among fourth grade students?

2. What strategies do fourth grade students utilize in solving word problems that require mathematical reasoning?

Theoretical Framework

Beginning with the work of Clark and Kamii (1996) we identified five levels of reasoning. The five developmental levels described below emerged from a review of the literature and provided the framework for this study. The multiplicative reasoning levels, described by Thorton and Fuller, 1981; Karplus and Lawson, 1974; Clark and Kamii, 1996, are summarized in Table 1 and provide a framework for this study.

Table 1.

Multiplicative Reasoning Levels

1. Spontaneous Strategy	Not yet additive/guessing
2. Additive Strategy	Derives answer utilizing addition or subtraction
3. Multiplicative Strategy (w/o success)	Cannot make transition from additive to multiplicative thinking, but understands additive is not sufficient
4. Multiplicative Strategy (w/ success)	Uses multiplicative reasoning successfully with time to reflect in describing the relationship between the numbers
5. Proportional Strategy	Introduces a new quantity as a unitizing factor then successfully completes problem through multiplication or division

(Adapted from Thorton & Fuller, 1981; Karplus & Lawson, 1974; Clark & Kamii, 1996)

Modes of Inquiry

Participants

In cooperation with the administration of a large urban school district in North Carolina, the participants were recruited from the fourth grade at a Chapter I school. The participants were age nine or ten and were recommended for study by the school's math specialist as being among the strongest math students in fourth grade. Overall fourteen participants of varying ethnicity, nine of whom were female and five of whom were male, were engaged in the study.

Instrument

The participants were asked to engage in a dialog while being interviewed. Because the computer can provide an environment that can enhance children's own construction of multiplicative reasoning via interaction with the teacher (Olive, 2000), the computer was employed as a research tool. An instrument consisting of ten questions was devised to invoke varying levels of multiplicative reasoning. Items were adapted from paper and pencil instruments developed by previous researchers (Thorton & Fuller, 1981; Karplus & Lawson, 1974; Clark & Kamii, 1996). The test instrument was developed in *Microsoft Visio* such that the participants were able to drag and drop green fish into the larger yellow fish to demonstrate their understanding of the multiplicative reasoning problems presented. Also provided were manipulatives that included a magnetic aquarium containing green and yellow fish, two pizza pans for test item nine, and manipulatives such as paper clips, buttons, and stick figures for participants to demonstrate their understanding of test item ten.

Data

The researcher employed two video cameras, one focused on the computer keyboard, computer screen, and hands of the participant, and the other focused on the participant's written workspace. In addition to the video recordings, the participant responses were collected by the Microsoft Visio program. The participants also provided written work supporting their thinking during the interviews. Transcriptions of the participants' verbal responses were also made. By reviewing the video recordings, in conjunction with the Microsoft Visio files, the written work of the participant and the transcriptions, the researchers explored indications of the emergence of participants' multiplicative reasoning.

Analysis

The data were examined for indications of the emergence of multiplicative reasoning

utilizing the following techniques. Participants' data were first coded with respect to the framework introduced in Table 1 by observing the manner in which the participants responded to each question. Coded selections were then reexamined for key words, similarities, and differences within each of the five levels. When analyzing the participants' data, it became apparent that there were indicators not found as descriptors in the original five levels identified in Table 1. These indicators were inserted into Table 2 and then the participants' data were reanalyzed using this expanded framework.

Results

The researcher placed the participants' key words, transcripts, utterances, written work, schemes, and drawings for each question into one of the twelve strategies based on the following criteria found in the indicators column in Table 2. If the participant exhibited non-preservation of the quantification of objects, then the participant was placed at Level 1 Non-quantifier. For example, if the participant placed 32 fish into a fish that could clearly not hold 32 fish, this would be an example of non-preservation of the quantification of objects. If the participant arrived at the answer through guessing, then the participant was placed at Level 1 Spontaneous Guesser. For example, a demonstration of Level 1 Spontaneous Guesser behavior was exhibited when the participant noted that a particular fish should be fed 7 fish because 7 was his favorite number.

When the participant specifically indicated a need to multiply because the problem had the word "times" (or some other keyword such as twice), the participant was placed at Level 2 Keyword Finder, because the participant derived the answer by invoking the multiplication algorithm. When the participant counted the fish and uttered answers where it was easily observed that the answers were related to a one-on-one mapping with the whole number system, the participant was placed at Level 2 Counter.

Table 2 presents the strategies employed by the participants on the research instrument. The reference column in Table 2 identifies experts in the field who agree that the indicators in column three signify the emergence of multiplicative reasoning. Table 2 assists in answering research question one: what are the indicators of multiplicative reasoning among fourth grade students?

Participants who arrived at their answers via addition, and plainly indicated so by writing or saying that they were adding more fish for each fish, were placed as a Level 2 Adder. Quite

often, because participants had success utilizing additive strategies, they tended to utilize additive strategies when they did not have a plan, perhaps because they had been successful in the past. However, utilizing an additive strategy demonstrated more understanding than simply guessing because their answers corresponded to the size of the fish. When participants made good use of the fact that $A < B < C$, then the participants were placed as a Level 2 Quantifier. When participants could not find the answer via multiplication they sometimes would measure the relative size of the fish, thus measuring the fish on the computer screen or the manipulatives and derive an answer through measurement. When the participants derived their answers via measurement, then the participants were placed as a Level 2 Measurer.

Sometimes the participants understood that additive strategies were not successful and attempted, but did not succeed, at utilizing multiplicative reasoning strategies. Often participants utilized repeated addition to obtain the correct answer. They would demonstrate this by writing or uttering $3 + 3 + 3 = 9$, for example. When such repeated addition was utilized, the participants were placed as a Level 3 Repeated Adder.

In many cases the participant may have understood that additive strategies were not sufficient and succeeded at utilizing multiplicative reasoning by demonstrating a good ability to coordinate the objects, numbers and operations defined within the word problem. Such participants often obtained the correct answer but did not or were not able to articulate the method or schema utilized to achieve the correct answer. Such demonstrations provided support for the participant being placed as a Level 3 Coordinator.

If the participant articulated an adequate mathematical sentence, either on paper or verbally, which fully described the mathematical relationship between the fish, the numbers, and the operations, then the participants would be placed as a Level 4 Multiplier. Additionally, some participants spoke or wrote the word "cut" or a similar word to indicate the need for division. In such a case, the participants were placed as a Level 4 Splitter.

When the participants provided a new quantity as a unitizing factor, articulated an adequate mathematical sentence, and successfully completed the problem using either multiplication or division, then the participants were placed as a Level 5 Predictor. In the case of Question 10, the participants exhibiting Level 5 Predictor should have stated something similar to "6 is to 4 as y is to 6 and since 6/4 reduces to 3/2 which equals 1.5 (this is the new unitizing factor) then $1.5 \times 6 = 9$".

Table 2.

Strategy Table for Level Mastery

Level	Strategy	Indicator	Reference
Level 1	Non-quantifier	Exhibits non-preservation of the quantification of objects, i.e. A < B < C, meaning that 7 can = 8, can = 9	Clark & Kamii, 1996
Level 1	Spontaneous Guesser	Arrives at answer through guessing	Clark & Kamii, 1996
Level 2	Keyword Finder	Derives answer from keywords such as times and applies the associated algorithm	Sowder, 1988
Level 2	Counter	Enumerates objects with a one-on-one mapping with the whole number system	Dienes & Golden, 1966; Steffe, 1988
Level 2	Adder	Derives answer utilizing addition or subtraction regardless if strategy leads to success	Nunes & Bryant., 1996; Steffe, 1988; Thompson & Saldanha, 2003
Level 2	Quantifier	Makes use of the fact that A < B < C	Clark & Kamii, 1996; Lamon, 2006
Level 2	Measurer	Exhibits an understanding of when measurement should be linear or curvilinear and that each measurement has a starting and ending point without overlap or gap between unit measures	Kaput & West, 1994; Karplus, Pulos, Stage 1983
Level 3	Repeated Adder	Demonstrated an understanding that multiplicative answers can be achieved through repeated addition	Fishbein, Deri, Nello, and Marino 1985; Nunes & Bryant, 1996; Piaget, 1965; Steffe, 1994; Vergnaud, 1983, 1988
Level 3	Coordinator	Demonstrates limited ability to coordinate objects, numbers and operations	Park & Nunes, 2001
Level 4	Multiplier	States a multiplication sentence and demonstrates fluency with respect to coordination of objects, numbers and operations	Clark & Kamii, 1996
Level 4	Splitter	Utilizes concept of cutting and halving to indicate the need for division	Confrey, 1994
Level 5	Predictor	Predicts the measure of an object in 1 system given the measure of a proportional or similar object in another system	Kaput & West, 1994; Lamon, 2007

The literature provides a clear baseline for understanding multiplicative reasoning outlined in Table 1. Yet the participant data reveal that a more detailed analysis can further identify the building blocks underlying multiplicative learning for fourth grade students. Table 2 identifies multiplicative indicators of these fourth graders along with additional strategies that expand the ideas in Table 1. These expanded strategies and indicators are then employed to re-analyze each of the 14 individual participant's performance on the ten item instrument. The columns in Table 3 explain the frequency of the strategies used by the participants. The rows indicate the individual participant's use of strategies. This analysis addresses our second research question concerning strategies.

Table 3.

Participant Strategies Frequency Table

Participant	Level 5 Predictor	Level 4 Splitter	Level 4 Multiplier	Level 3 Co-ordinator	Level 3 Repeated Adder	Level 2 Measurer	Level 2 Quantifier	Level 2 Adder	Level 2 Counter	Level 2 Keyword Finder	Level 1 Spontaneous Guesser	Level 1 Non-quantifier
1		2	6	1				1				
2			1		2	2		2	1		2	
3	1		5	1				1			2	
4			3				1	3	1	1	1	
5			3			1		2	1	2		1
6			3		2	2				3		
7			5		1	2				2		
8		1	3	1	1		1	2	1			
9			4	1		3	1	1				
10		1	5			2		2				
11	1	1	7			1						
12			2	1		2		4				1
13	1		5	1		1				2		
14		3	2	1		1				2		1
Σ	3	8	54	5	8	17	3	18	4	12	5	3

Table 4 is depicts the item analysis frequency with respect to each of the 10 test items. The columns in Table 4 explain the frequency of the strategies invoked by each test instrument question. The rows indicate the number of participants that used the particular strategy.

Table 4.

Item Analysis Frequency Table

	Level 5 Predictor	Level 4 Splitter	Level 4 Multiplier	Level 3 Co-ordinator	Level 3 Repeated Adder	Level 2 Measurer	Level 2 Quantifier	Level 2 Adder	Level 2 Counter	Level 2 Keyword Finder	Level 1 Spontaneous Guesser	Level 1 Non-quantifier
1			9		3		1			1		
2		4	6	1	2					1		
3			11							3		
4			3		2	1		5		3		
5		2	3	1				6		1	1	
6			7		1			3	1	1	1	
7			8				1	1			1	3
8		1	6	1				2		2	2	
9	2	1	1	2		7	1					
10	1					9		1	3			
Σ	3	8	54	5	8	17	3	18	4	12	5	3

Examining the results reported in Table 3 and 4, we observed the frequency of individual participant's strategy use. It was interesting to note the spread of strategies and indicators occurring over these fourth-grade participants. Therefore we sorted the participants' strategies into three categories. Those who were consistent in their choice of level 4 strategies or above (more than 70% of the time) we label as multiplicative reasoners. Pre-multiplicative are those participants who consistently chose strategies below level 4 in their attempts to solve the problems. Those participants who used a range of strategies we label as emergent. Emergent participants are considered non-multipliers. Table 5 summarized these findings.

Table 5: Multiplicative Reasoning Strategies

Pre-Multipliers (7 times out of 10)	Emergent (5/5 or 4/6)	Multipliers (7 times out of 10)
Participant 2	Participant 3	Participant 1
Participant 4	Participant 7	Participant 11
Participant 5	Participant 8	
Participant 6	Participant 9	
Participant 12	Participant 10	
	Participant 13	
	Participant 14	

Summary and Implications

The indicators of multiplicative reasoning are specific writings, utterances of keywords, and

behaviors of these participants as they engaged in problem solving. From these indicators 12 strategies were identified that these fourth grade participants utilized in solving multiplicative reasoning word problems. It was interesting to note that each of the 12 strategies was used at least three times. Separating the participants into three categories provided additional insights into the vast spread of multiplicative understandings present in Chapter I schools in fourth grade. In the future, we propose to investigate the strategies and indicators of older children and younger children with the research instrument to provide validation for our findings reported here.

The practical implication of our findings is that they provide teachers with another tool to diagnose and assess students' understanding of multiplication. Teacher observations and informal assessment techniques can be used in conjunction with the list of indicators generated in Table 2. This list is another way for teachers to assist students' progress towards understanding multiplication. The list of indicators and strategies is also useful in determining the trajectories of students' multiplicative reasoning so that teachers can remediate and develop steps towards additional understanding (Ball, 2003; Richardson, Berenson, and Staley, 2009).

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ENACTMENT OF JUSTIFICATION TASKS IN ELEMENTARY SCHOOLS: A MULTI-CASE STUDY

Jessie C. Store   Sarah B. Berenson   Nancy T. Payne
The University of North Carolina at Greensboro
jcstore@uncg.edu   sbberens@uncg.edu   ntpayne@uncg.edu

Standards for mathematics require teachers to employ teaching practices that promote justification of mathematical ideas. This expected teaching practice is situated in substantial research on students' difficulties to justify mathematical ideas. In exploring these difficulties, research tends to focus on conceptions of justifications or proofs. However, Bieda (2010) recommends that, to better understand students' practices in justifications, there is a need to understand how teachers who have undergone professional development enact justification tasks. As part of the ON TRACK project, this multi-case study reports practices of 3 elementary school teachers in enacting justification tasks.

Russel, Schifter and Bastable (2011) observed that, despite emphasis on making and justifying general mathematical claims in curriculum standards (e.g., The National Council of Mathematics Teachers 2000 standards), students tend to conclude that particular mathematical claims work out for all instances after finding just a few examples that support the claim. For example, Healy and Hoyles (2000) reported high school students' tendency to use example based arguments when working on justification tasks. Ellis' (2007) and Lannin's (2005) studies with middle school students reported the same tendencies. Drawing from Stein, Grover and Henningsen's (1996) definition of mathematical task, we define a justification task as a set of classroom activities that aim at developing students' justifications about general mathematical ideas conjectured in pattern finding activities.

To shed light on students' difficulties in justifying mathematical arguments, Knuth (2002) and Kuchemann and Hoyles (2009) studied teachers' and students' conceptions of proof respectively. These studies show that empirical justifications are favored. Very few studies have attended to instructional practices (Bieda, 2010). Furthermore, Bass (2011) wrote that, in the few cases that studies focused on instruction of justification and proofs, the focus was normally on teachers' knowledge and beliefs. Other studies have focused on curricular goals in teaching of justifications (e.g., Richardson, Carter & Berenson, 2010) and recommended instructional moves (e.g., Komatsu, 2010). Thus, we still do not know enough about how elementary school teachers enact justification tasks.

Bieda (2010) recommends that, to better understand students' practices in justifications, there is a need to understand how teachers who have undergone professional development enact

justification tasks. Our objective for this study therefore, is to explore how elementary school teachers who underwent professional development about teaching reasoning tasks and were provided with recommended instructional moves enacted justification tasks.

Conceptual Framework

We employed Stein et al.'s (1996) conceptual framework to study how teachers enacted justification tasks. This framework accounts for instructional materials, teachers' set up of the task in the classroom and implementation of the task by students as contributing factors to students' learning of the intended mathematical ideas.

Task set up

Task set up refers to how the teacher launches a task. This may include a discussion of what is expected and distribution of tools for carrying out the task. Task set up may alter cognitive demands, the extent to which students use multiple representations, solution strategies, and the type of justifications that will be pursued. Factors that affect task set up include teacher's goal for the task and teachers' knowledge.

Task implementation

Task implementation involves how students work on the task. It involves students' engagement in using multiple representations and the extent to which they engage in mathematical argumentation and justify mathematical conjectures. Teachers' pedagogical and students' learning behaviors, and tasks' attributes affect task implementation. Researchers interested in task implementation may ask if students are actually engaged in the critical features of the tasks as was intended in the curricular materials.

Research studies that have drawn from this framework include Bieda (2010) and Stein et al. (1996). Bieda (2010) studied enactment of proof related tasks with middle school teachers and students and focused on these parts of the framework: (1) mathematical task as written, (2) teacher factors influencing task set up and (3) student learning. Stein et al. (1996) focused on task set up, implementation and factors affecting implementation in grades 6 through 8 mathematics classrooms. Our present study was conducted with elementary school teachers and focuses on pedagogical practices in enacting justification tasks. Therefore, as figure 2 shows, our focus is on features of the written task, features of the set up task and teachers' instructional habits. We also focus on student learning. Our central research question was; how do teachers enact justification tasks? To explore this question, we further asked

1. What task features were represented in instructional materials?
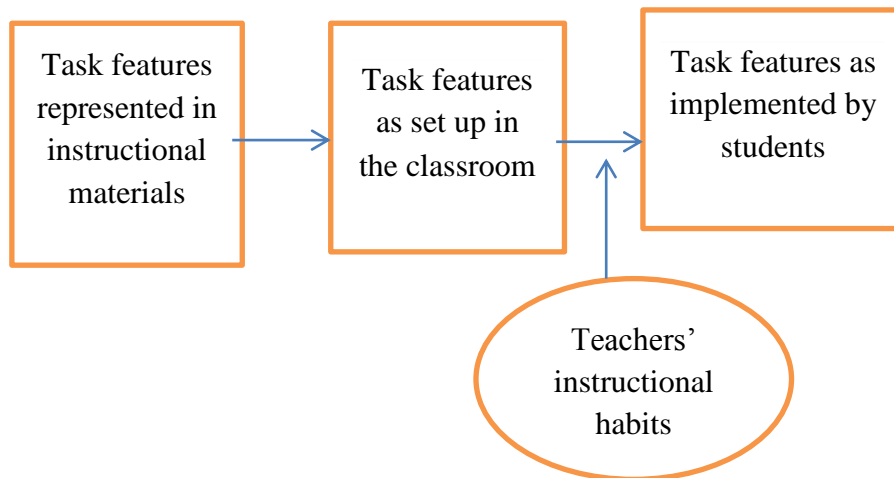2. What task features were enacted in the classroom?

Figure 1: A framework for analyzing enactment of justification tasks.

Methodology

Context

This study is part of the On Track Learn Math project. This project aims at fostering elementary school students' mathematical reasoning through engagement in pattern finding activities in after school enrichment programs. On Track elementary school teachers participated in 3 five week blocks of professional development that focused on developing mathematical reasoning, content knowledge, knowledge of students' reasoning and productive pedagogical practices. Professional development participants met once a week for 90 minutes. During professional development meetings, participants worked on the same mathematical tasks as they used in On Track classrooms, experienced, and discussed the intended pedagogical practices. Teachers were given instructional materials that included lesson plans to guide their enactment of On Track curriculum. For this study, we report practices of 3 teachers whose general teaching styles seemed representative of the different styles of all the 12 On Track teachers. Their elementary school teaching experience ranged from 6 to 22 years.

Data collection

Data for this study were collected after the teachers attended the first professional development session. We used video cameras to record classroom activities of the teachers and the students. The video cameras focused on the teachers and students during whole group and

small group discussions. Each class had about 15 students. Students' written artifacts were also collected at the end of each lesson.

Mathematical problem

Students were expected to find the maximum number of people that can sit around a train of one, two, three, 10 and 100 train tables if one person sits on each side of the small table making up the train (see figure 2). They were asked to observe patterns, make and justify conjectures for number of people for n-table train.

Figure 2: Train of Square Tables

Data analysis

The framework on enactment of tasks guided our data analysis. We conducted a content analysis of the instructional materials to identify intended features of the On Track curriculum. According to Matsumura, Garnier, Slater and Boston (2008), three raters are sufficient to rate quality of instruction. On a scale of 0 to 4 (1 for not enacted, 2 for barely enacted, 3 for almost adequately enacted and 4 for adequately enacted), three raters independently assessed the extent to which the teachers implemented the features of the intended task. All raters attended professional development with teachers and participated in discussions of ideal practices for enacting justification tasks in elementary classrooms. Qualitative methods were used to analyze practices of each case and identify themes.

Results

Content analysis of On Track instructional materials showed several features that aimed at developing and nurturing students' justification schemes. For this paper, we focused on enactment of features we categorized as creating or taking up opportunities to (1) ask for justifications and (2) develop different justification schemes. We first present enactment across cases followed by each case.

Enactment across Cases

Creating or Taking up Opportunities to Ask for Justifications

Intended features

We identified three features in this category. Teachers were expected to ask students for convincing arguments about validity of their rules. In addition, they were expected to ask

students to evaluate peer's responses and question each other. Teachers were expected to use student's conflicting responses as opportunities for students to justify their rules to each other.

Enacted features

Three raters observed the lesson and assessed the extent to which each of the teachers enacted these intended features. Average ratings of the extent to which each teacher enacted task features are presented in Table 1. From the ratings, while one teacher adequately asked students what they thought about their peers ideas and strategies, the other teachers did not. Using conflicting responses as an opportunity for students to justify to each other was the least enacted feature.

Table 1.

Average Ratings on How Teachers Took up or Created Opportunities to Ask for Justifications

The extent to which teachers took up or created opportunities to :-	**Teacher 1**	**Teacher 2**	**Teacher 3**
Ask why and why not questions	2	1	1
Ask students what they think about peers' answers	3	.6	.6
Use conflicting responses as opportunities for students to justify to each other	1	0	0
Ask students to question each other	1.33	.33	1

Creating or Taking up Opportunities to Develop Different Justification Schemes

Intended features

On Track instructional materials encouraged teachers to support students' different ways of justifications. Teachers were encouraged to support students' analytic justifications that referred to the general context of the mathematical problem by asking students to explain how parts of their rules connected to the square table train problem. Teachers were discouraged from being judges of validity of responses. Rather, teachers were supposed to ask students to convince each other of validity of their ideas. Thus, authoritarian schemes were discouraged.

Enacted features

As intended, teachers' (2/3) instructional habits did not at all encourage students to refer to others as sources of validity (see Table 2). Empirical justifications were the most enacted schemes. On the other hand, analytic justifications were barely enacted.

Table 2.

Average Ratings on How Teachers Took up or Created Opportunities for Students to Develop Different Justification Schemes

The extent to which teachers took up or created opportunities for students to:-	Teacher 1	Teacher 2	Teacher 3
Develop analytic justifications by connecting the rule to the model (e.g., ask where does the +2 in 2n+2 come from?)	1	1.33	0
Develop empirical justifications (e.g., nurture example based justifications or actual counting of seats using manipulatives or drawings)	2.66	1	2
Develop authoritarian justification schemes (e.g., nurture "it is true because my teacher said so" or "because that's what multiplication does.")	0	1	0

Enactment by Each Case

Teacher 1

Teacher 1 asked "why" questions and her students expected their peers to justify their conjectures. For example, she asked a student to "prove (that 202 people would sit around 100 square tables) without drawing 100 tables." When this student hesitated, the other students chanted "Prove it! Prove it! Prove it!" Justifying by counting the actual seats was discouraged but example based justifications and justifications by drawing the general context of the problem were privileged. After 2 students justified their rules in different ways, Teacher 1 said, "So, it looks like his (student's) rule works. And he proved it. He proved it with kind of a picture in mind, right? (She) proved it (her rule) with a formula in mind (by trying out 3 examples)." However, students tended to use a few examples as a sufficient justification and in such cases, the teacher did not push students to justify if their rule works for *any* number of tables. Teacher 1 rarely asked students to question each other, but she often asked students to evaluate peers' rules by applying them to check if their strategies yielded similar results.

Teacher 2

Teacher 2 seldom asked students to justify their conjectures. She sometimes said to the students: "make sure you explain. There it says convince me your rule works. Don't just say I know it works." In this classroom, asking for a justification had a form of "explain how you got your answer." Teacher 2 did not directly ask students to question each other, but encouraged students to explain their conjectures to peers in ways that peers would understand. Additionally,

after the task was launched, students worked the problem out and wrote down their responses, whole class discussions did not follow. This lesson design did not take up any opportunities to use conflicting responses for justification.

Teacher 3

Teacher 3 asked for justifications in both small and whole group discussions. In episode 1, students were justifying conjectures on how many people would sit around 100 and n tables. When students shared conflicting responses, Teacher 3 praised student thinking but did not take up the opportunity for students to justify to each other (see episode 1). For correct responses, Teacher 3 sometimes asked students if their peers' responses were correct. The class was content with example based justifications.

> **Episode 1**
>
> Student 1: I just multiplied 100 by 2
>
> Teacher 3: So why did you multiply 100 by 2
>
> Student 1: Because my rule was multiply by 2
>
> Teacher 3: You are thinking (calls student 2 to present his rule)
>
> Student 2: My rule is x 2 +2 (writes on the board) because 1 x 2 equals 2, plus 2 equals 4.
>
> Teacher 3: Ok try the next one (input).
>
> Student 2: 2 x 2 equals 4, plus 2 equals 6 and then 3 times 2 equals 6, plus 2 equals 8.
>
> Teacher 3: Ok. So how about somebody who said 100 tables equals 202 people. Was that right then?

Discussion and Conclusion

Although the teachers' sample size and the lessons observed were small, this study brings to our attention important issues in mathematics education. It shows different ways justification tasks may be enacted in elementary classrooms. In general, enacting justification tasks is both a pedagogical and content problem for teachers. The mathematics education community is challenged to think of ways to use strengths of different pedagogical approaches to develop stronger expertise to enact justification tasks. This study reveals teachers' tendency to encourage and accept example-based justifications as sufficient. This aligns with previous research (e.g. Bieda, 2010). However, contrary to views that teachers usually position themselves as sources of knowledge thereby instituting authoritarian schemes as acceptable, these teachers did not. This

study also brings about an important question for research and practice: how can we support teachers to enact curriculum that requires students to develop justification fluency?

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SECONDARY STUDENTS' MULTISTEP PROBLEM SOLVING IN STEM CONTEXTS

Sarah Johnson
UNC Charlotte
SJohn276@uncc.edu

David K. Pugalee
UNC Charlotte
David.Pugalee@uncc.edu

A Science, Technology, Engineering, and Math (STEM) curriculum was developed for high school students to encourage their engagement in mathematics through problem-based applications. The STEM curriculum, Mathematics Investigations Using Design Science and Engineering Tools (MINDSET), is funded by the National Science Foundation and is intended as a fourth year secondary course. The student participants within this research study, who took pre- and post-test assessments, demonstrated their ability to perform multi-step problem solving and find results in problem contexts. The findings suggest that the MINDSET curriculum gave students the ability to become competent in mathematical problem solving through practical application.

Over the past decade, the nation's political leaders have become concerned with the United States relinquishing its competitiveness in science, technology, engineering, and mathematics (STEM) (Kuenzi, 2008; NSF Report, 2007). Congress concluded that there is a growing concern that the United States is not preparing a sufficient number of students, teachers, and practitioners in the areas of STEM (Kuenzi, 2008). In addition, when Congress evaluated the proportion of natural science and engineering degrees obtained by 24-year-olds in the United States, it was revealed that in 2008 the U.S. was ranked 20th in comparison to all other nations (Kuenzi, 2008; Zhe, Doverspike, Zhao, Lam, & Menzemer, 2010). In order to stop the downfall of a nation that was based on innovative principles, interventions have to provide an opportunity for the country's future generation to extend their knowledge and understanding of mathematical content and problem contexts indicative of STEM disciplines (Subotnik, Tai, Rickoff, & Almarode, 2010).

To encourage students' pursuit in STEM education and careers, the government has endorsed several programs supported by the National Science Foundation (NSF) and National Aeronautics and Space Administration (NASA) (NSF Report, 2007). This research study examines students' understanding of math content through problem-based applications as a result of participation in the NSF funded program, Mathematics Instruction using Decision Science and Engineering Tools (MINDSET) (Chelst, Edwards, Keene, Norwood, Pugalee, & Young, 2011). MINDSET was developed to improve math students' abilities to formulate and solve multi-step problems and interpret results, while improving students' attitudes towards mathematics (Chelst et al., 2011). The MINDSET curriculum was designed to help students engage in mathematics through

real-world based problems. Incorporating a curriculum that exercises problem-based learning and visual demonstrations will assist students in appreciating mathematics and encourage them to consider further studies in STEM areas (Chelst et al., 2011).

The way students learn mathematics plays a major part in how they perceive it. As a result, practitioners are becoming more strategic in how they foster creative mathematical thinking in the classroom, through the implementation of real-world based problems. According to Freudenthal (1973), mathematics should be perceived as human activity. Through this positive perception, students can relate math to their everyday life. The purpose of this study is to examine the relevance of problem-solving by analyzing students' scores in pre-and post-test assessments that focuses on problems based in real-world contexts.

Problem-solving is considered to be a major component in mathematical leaning. This component of mathematics allows the students to learn math content while simultaneously developing thinking and reasoning skills. According to Blum & Niss (1991) there are two categories of problem solving: applied problem solving and purely mathematical problem solving. Both categories, which are complementary to one another, contain content and structure. In applied mathematical problem solving, a model conception is utilized as a way for conditions and developments to be made in order to achieve an adequate solution (Blum & Niss, 1991; Zandieh & Rasmussen, 2010). Students are utilizing contextual information in order to facilitate the construction of the problem. In regards to pure mathematical problem solving, basic concepts that lead to practical application become an imperative focus. The distinct terminology associated with purely mathematical problems solving is emphasized in order to learn mathematical concepts and the critical skills to solve problems. When students begin to learn mathematics within their formal schooling, they are taught the practical ways to solve equations and other mathematical problems. However, in order for students to see the relevance of mathematics, they are given applied mathematical problems. These forms of analytical and numerical methods allow students to become more versatile in various disciplines, such as engineering, science, business, and economics (Langdon et al., 2011; NSF Report, 2007).

The Curriculum and Findings on Student Problem Solving

The MINDSET curriculum consists of two sections: deterministic and probabilistic. Problem contexts covered in the deterministic curriculum include linear programming, the critical path method, facility location problems, and transportation problems. In the probabilistic section,

topics include probability distributions, decision trees, quality control, and queuing theory. Techniques and tools incorporated in MINDSET encourage students to utilize logical and deductive reasoning to solve problems (Chelst et al., 2011). One technique MINDSET incorporates in solving problems is reading and discussing the problem context. Within this approach, students funnel their understanding through multiple questions: Where do the numbers come from? What do they mean? What is the objective? What are the constraints? By using this particular technique, students are able to identify solutions or errors in order to solve multi-step word problems. The participants in this study were high school students in grades eleven and twelve. Thirty-four students were enrolled in the class; however, only twenty-six students completed both the pre- and post-test assessments. The demographics of the students in the study are summarized in Table 1.

Table 1.

Demographics for students participating in the pre- and post- test assessment

	Gender		Ethnicity		
# of students	Female	Male	White	Black	Hispanic
26	9	17	10	12	4

   The pre- and post-test assessment consisted of 28-items in the following areas: number sense, compound inequality, and proportions. When developing the pre- and post- test assessment, project personnel developed items that reinforced students' abilities to enhance their mathematical knowledge and skills by formulating and solving multi-step problems and interpreting results. Each item had a scoring rubric to guide the grading process. 68% of the items were based on a 0 and 1 scale. 29% of the items were based on a 0, 1, 2 scoring rubric. Within the items rated 0, 1, or 2, students were required to show their work. If they did not show their work, they would not receive full credit. There was only one item that was based on a 0, 1, 2, and 3 scoring rubric.

   Before scoring the MINDSET pre- and post- assessments, there were two pilot studies conducted to determine the adequacy of the scoring rubric. The pilot studies helped reduce the occurrence of scoring discrepancies. After constructing a final draft of the rubric, two scorers

assessed the pre- and post- assessments. As a result, students' assessment scores were consistent.   For the purpose of this exploratory study on the pre- and post- assessment, we selected questions that students displayed the most improvement. However, this does not discredit the improvement students attained from the other assessment questions.

In this discussion, certain items will be provided to show the setup of the problems. Table 2 shows those problems for which there was an improvement greater than 40%. These problems are described in the following section. Analysis of the assessments demonstrated that students were able to provide step-by-step explanations which demonstrated their mathematical reasoning. This suggests that students' elaborations on their answers reaffirms the confidence they gained after participating in a full semester of the MINDSET curriculum, which deals with a variety of application word problems requiring practical mathematical reasoning.

Assessment Items

Table 2.

Problems with Positive Changes of 40% or Greater in Student Performance

Test Question	Problem Context	Percentage of Improvement	Problem Content
Question 2b	Nutrition: Minimizing the number of calories for an athlete while meeting the basic nutritional requirements	42.3%	number sense
Question 2c	Nutrition: Minimizing the number of calories for an athlete while meeting the basic nutritional requirements	46.2%	Number sense
Question 6a	A student organizing his or her college class schedule	42.3%	Optimization problems
Question 7	Camp director organizing the living arrangements of a children's camp	46.2%	Proportions and Optimization problems

Question 2

The first part of question 2 pertains to selecting a recommended calorie intake from charts which display caloric information for adults and activity level by various occupations. The second part requires students to use number sense to decide if certain meal choices will allow a

diner to stay within +/- 120 calories of their diet plan. The third part of the question also requires number sense as students have to analyze whether a different choice of menu items would conform to the parameters. Within these questions, students have to use a compound inequality to address these parameters. Student's response in Figure 1-1a demonstrates that there was a lack of conceptual understanding in which he or she received a zero. However in Figure 1-1b, the student received a two, the highest score attainable, by demonstrating his or her complete understanding of the problem.
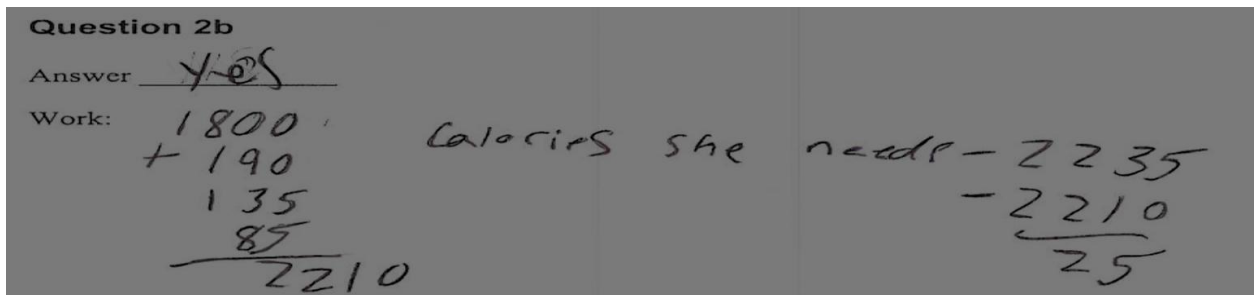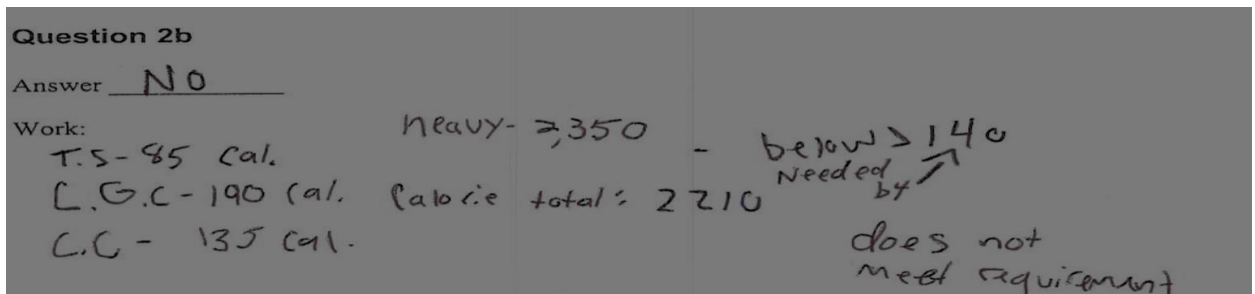
Figure 1-1a: Pre-Assessment Response

Figure 1-1b: Post-Assessment Response

Question 6

A college is offering 12 subjects for a three-year course of study with each subject spanning a year. Presented with certain constraints, students must specify which subjects must be offered for which year in order to meet all the constraints presented for the program such as each student taking four subjects per year and completing the degree in three years. This problem requires students to analyze a chart and delve into possible combinations that allowed them to achieve a goal. The students must follow certain mathematical constraints to ensure the appropriate plan is devised. A student's response (Figure 1-2) received a three on the post-assessment which is the highest score achievable. This student demonstrated an in-depth understanding of the concept

and content of the problem. The pre-assessment response was not provided because the student did not attempt the problem in which he or she received a zero.

Figure 1-2: Student's response from post-assessment question 6

	Subject 1	Subject 2	Subject 3	Subject 4
Year 1	Mechanics 1	Business 1	Computer 1	Technology 1
Year 2	Mechanics 2	Business 2	Computer 2	Electronics 1
Year 3	Technology 2	Business 3	Computer 3	Electronics 2

Question 7

In question 7, the problem requires students to follow parameters by analyzing a chart. In addition to analyzing a chart, the problem requires students to create certain combinations, which addresses proportions and the implementation of tree diagrams. The context is a camp in which 46 children and 8 adults must be housed in seven dormitories with various capacities. Presented with 4 rules that must be met in placing residents in each dorm, students have to develop a plan for placing the campers in dorms.

Conclusion and Implications

This study analyzed the improvement made in students' problem solving skills as a result of their involvement with the MINDSET curriculum. These findings suggest that, in general, students improved in their ability to solve problems based on logic and deduction. The data suggests that students continued to extend their problem solving skills through the implementation of MINDSET.

This study further affirms that when students master the processes of problem solving, they are able to hurdle over the complexity of mathematical application and mathematical modeling. Applied problem solving is extremely relevant; through this perspective, students are able to see the relevance of mathematics in everyday life (Blum & Niss, 1991). Through STEM-based curricula, real-world problem solving assists students in seeing the need for mathematics. Authors of STEM curricula have to be strategic in how they develop problems by making sure each problem has strong math content and maintains students' interests. Within STEM curricula, students learn how to communicate and become engaged in learning about STEM concepts and

skills (Chelst et al., 2011; Subotnik et al., 2011). Being able to learn math content through the problem-solving process should be a core component of any successful mathematics problem.

Further investigation and experimentation into the implementation of the MINDSET curriculum is necessary. This study only assessed one classroom, with a limited number of students which may limits the interpretation from the results   Future research should therefore concentrate on the investigation of at multiple classrooms that implement MINDSET and classrooms that do not.

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AN INVESTIGATION OF THIRD GRADE STUDENTS' GENERALIZATIONS USING FUNCTION MACHINE TASKS

Nancy Tilley Payne Sarah Berenson Jessie Store

The University of North Carolina at Greensboro

ntpayne@uncg.edu sbberens@uncg.edu jcstore@uncg.edu

The National Council of Teachers of Mathematics (2000) content standards advocates that algebra should be taught at all grade levels. Teachers can help students improve their algebraic skills by using relevant tasks and instructional practices that elicit students reasoning. One type of task that promotes algebraic reasoning is a function machine. This study looked at the types of generalizations student wrote to describe patterns in input/output tables. The results showed that third grade students could successfully write explicit rules for both linear and quadratic data.

Recognizing a need for mathematics reform, The NCTM (2000) content standards advocates that algebra should be taught at all grade levels with the goal to develop students who are capable of reasoning, problem solving, communicating their ideas and using proof. Nevertheless, international studies have shown that there still exists a gap between students in the United States and their peers, in countries with similar economics and development as the US, as student's progress through school (NCES, 2007; NCES, 2004). Researchers are advocating introducing algebraic concepts in early elementary school in conjunction with arithmetic; not as a separate subject (Brizuela & Schliemann, 2004; Kaput, 2008; Warren, Cooper & Lamb, 2006). While algebraic reasoning encompasses many concepts, functional reasoning is one aspect of algebraic reasoning that is critical to students' success in higher level mathematics (Brizuela & Lara-Roth, 2002; Warren, 2005). The research suggests that in order to develop functional reasoning educators need to begin the process early by encouraging specific types of reasoning and allowing students to engage in tasks that require them to look for patterns in data and make generalizations based on those patterns (Blanton & Kaput, 2005; Confrey & Smith, 1994, Warren, 2005).

The research suggests that in order for students to develop functional reasoning, mathematical tasks should include opportunities for students to look for patterns in data. Input/output tables are particularly helpful in organizing student data and are often used in higher level mathematics. There are two ways students generally look at data, either vertically (down the input/output table) or horizontally (across the input/output table). Lannin, Barker and Townsend (2006) classified generalizations into two groups, recursive and explicit. A recursive

generalization describes the change in the dependent variable (output value) and uses the value of one term to find the value at the next term. In contrast an explicit generalization is a rule that allows for direct calculations of any output given its input by describing how the input relates to the output. According to Bezuszka and Kenney (2008) generalizing sequences using recursion has its limitations. First, it can be difficult to find a rule when the sequence is out of order and second, finding output values for large numbers can be problematic. If the input number is significantly large, students cannot find a way to proceed and learning is impeded. Explicit rules on the other hand, are more efficient since they can find output values for large input values but can be difficult for students to develop. However, finding explicit rules help students develop an understanding of functions.

In developing functional reasoning students must notice how the quantities are changing in relation to each other (NCTM, 2000). Many studies suggest that students generally have little difficulty in determining how a single data set changes but are challenged by how two data sets change in relation to each other (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Warren, 2005; Warren & Cooper, 2008). Warren (2005) studied young student's abilities to generalize rules for growing patterns and found that while students have the ability to reason about functions single variational reasoning (how one date set changes) was cognitively easier for younger students. Likewise, Lannin (2005) found that while working on patterning tasks, students often has less difficulty finding recursive rules but experienced difficulty moving from a recursive rule to an explicit rule.

Function Machine Tasks

Function machines can be helpful tools in developing students' conceptual understanding of functions. According to McGowen, DeMarios, and Tall (2000), these types of tasks provide a strong cognitive representation of the relationship between input values and output values. These researchers also argue that through the use of a function machine the domain and range can be introduced "naturally" as a set of possible inputs and outputs. Students also seem to be able to develop the idea of a specific domain mapping to a specific range. Another benefit of using a function machine is that it appeals to a wide range of student ability levels. In a study of students in a developmental algebra course, Davis and McGowen (2002) found that students felt that a function machine assisted them in making sense of notation, helped to organize their thinking and allowed them to produce equations.

While previous research suggests that function machine tasks promote the development of functional reasoning it also suggests that students experience difficulty writing explicit rules, especially students in early elementary school. With this in mind, the purpose of this study was to investigate the types of generalization rules that third grade students wrote while working on function machine tasks. The research questions for this study were 1) what types of rules do third grade students write when working with function machine tasks? 2) what types of tasks are the third grade students more likely to write explicit rules?

Methods

My study is taken from a larger study on students' algebraic reasoning in an after school enrichment program entitled On Track Learn Math. This program was conducted in four, five week sessions held in both urban and rural settings. The data collected for this study was taken from the second and third sessions of the program. Each session was introduced with a function machine task. These tasks allowed teachers to introduce important concepts such as generating data, using an input/output table and describing patterns. After students worked on the function machine tasks they were given a more open ended functional reasoning task or a combinatorical task. This study looks specifically at how the third grade students reasoned about the function machine task.

Participants

There were six total schools who participated in the On Track program. The student participants self-selected to attend the program and were chosen on a first come/first serve basis. Participation was limited to 18 students per school. The participants in this study ranged in age from seven to ten years (third through fifth grade), and were diverse in gender, ethnicity and academic abilities. In addition, three of the schools were considered Title 1 schools. The third grade students made up the largest population in this study.

The Function Machine Tasks

These tasks asked students to generate data, organize the data in an input/output table, look for patterns in the data and write generalizations that would provide an output value for a very large input value (e.g., 100). The functions that each machine generated varied in number of operations and types of rules (linear and quadratic). In addition, the function machine tasks were introduced in different ways. Figure 1 is the first function machine task the students worked on.

It was presented in graphic form. This machine shows how the input goes into the machine and the output comes out of the machine.
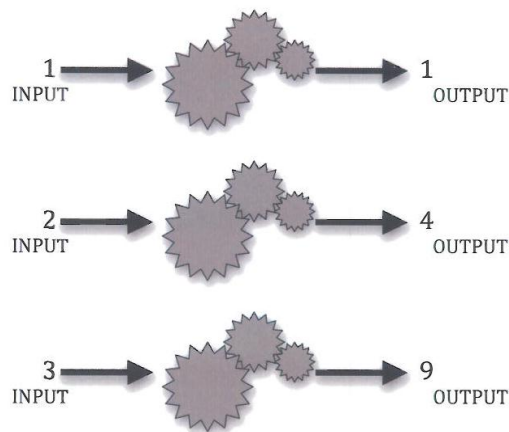

Figure 1: On Track Function Machine Task 1.

The graphic representation in figure 1 shows students the input and output so they can directly copy their values into their input/output charts. As the task progressed, the function machines were presented verbally (see figure 2) so the students had to make the decision about which value was the input and which was the output.

You rode your bike for one hour and the function machine said you had traveled 3 miles. When you rode your bike for 2 hours, the function machine said you had traveled 6 miles. How far had you traveled after 3 hours? _____

Figure 2: On Track Function Machine Task 3

Data Collection

The data was collected over two five week sessions of On Track. Student work samples from each session were collected and each session was videotaped. This study uses the student work samples as data. There were a total of fourteen function machine tasks used in this study.

Analysis

This analysis began with an examination of the type of generalizations the students made of the function machine rules. Codes were based on the Lannin, Barker and Townsends (2006) definitions of recursive and explicit generalizations. The student work samples revealed generalizations that fell into three categories. Table 1 outlines the codes that were used.

Table 1.

Student Rules Coding Scheme

Code	Meaning	Example
0	No response, incorrect rule	
1	Recursive Rule	The output increase by 3
2	Explicit Rule	The input x the input equals the output

After all of the student work samples were coded, the percentage of each type of generalization was calculated in order to determine how the students performed on these tasks (see Table 2).

Results

The analysis of the data found students tended to write explicit rules more often than they wrote recursive rules. In fact, at least 50% of the students wrote explicit rules for 8 out of 14 tasks and at least 70% wrote explicit rules for 4 of the 14 tasks. The students experienced overwhelming success on the one operation and linear function rules. They experienced more difficult with the more complex quadratic rules (task 10 &11). For these tasks the student tended to write a recursive rule to describe their patterns.
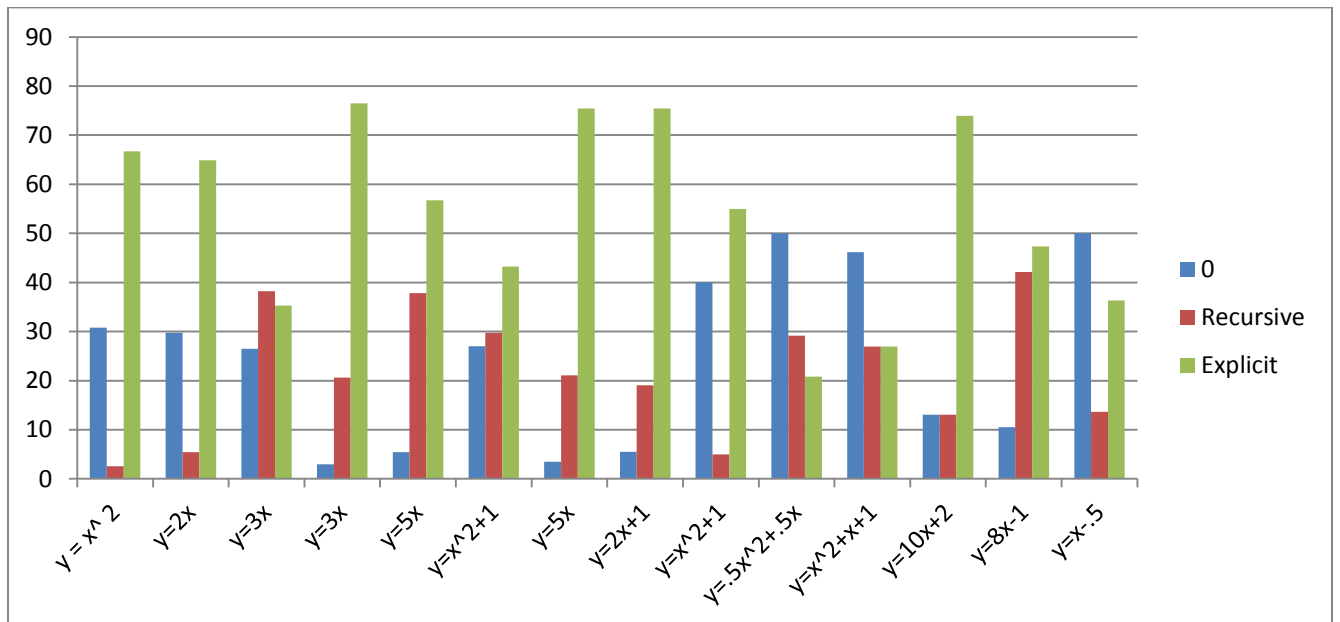

Figure 3: Percentage of Third Grade Students Generalization by Type

Discussion

The findings of this study agree with others (Blanton & Kaput, 2004; Carraher, Schliemann & Schwartz, 2008; Martinez & Brizuela, 2006; Lannin, 2005; Warren, 2005; Warren & Cooper, 2008; Warren, Cooper & Lamb, 2006) that students are capable of reasoning about functions at an early age. In addition, the ability to generalize rules is enhanced through the use of input/output tables (Smith, 2008; Warren, Cooper & Lamb, 2006) and function machine tasks. The types of rules (linear versus quadratic) and the number of terms played a role in the development of their rules. Students were more successful writing rules for linear or quadratic functions with one or two terms. However, they experienced more difficulty writing explicit rules when the quadratic rule contained three terms and when there was a fractional coefficient.

The students experienced The students experienced difficulty writing an explicit rule for function machine 3 which was a linear one term rule. It is important to note that the first two function machine tasks were represented by a graphic (see Figure 1). However, function machine 3 changed the representation to a verbal description (see Figure 2). The change in the type of representation may have lead students to see a recursive rule over an explicit one. For function machine 10, the fraction presented difficulties for the students. Since the students are in third grade it was expected that this rule would prove to be difficult for them. Lastly, function machine 11 was significantly difficult for the third grade students since it was a quadratic rule with three terms.

Previous research on students' generalizations found that developing explicit generalizations is difficult for young students and students tend to use recursive reasoning when analyzing patterns in input/output tables (Warren, 2005). This study does not support this claim. In fact, it supports the opposite that given appropriate tasks students as young as third grade can use functional reasoning to generalize explicit rules.

Implications

The findings of this study have implication for curricular changes in early elementary school mathematics. Functional reasoning is the basis of higher level mathematics (NCTM, 2000) and it takes years for students' functional reasoning to develop (Warren, Cooper & Lamb (2006). By beginning that process early, students have a greater chance of success in mathematics later in school. This investigation suggests that students should work with function machine tasks that use a variety of function rules and representations.

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EARLY STUDENT DISSONANCE IN INTRODUCTORY STATISTICS

Melanie Autin
Western Kentucky University
melanie.autin@wku.edu

Summer Bateiha
Western Kentucky University
summer.bateiha@wku.edu

Hope Marchionda
Western Kentucky University
hope.marchionda@wku.edu

Traditional classroom instruction consists of teacher-centered learning in which the instructor presents course material through lectures. A recent trend in higher education is the implementation of student-centered learning in which students take a more active role in the learning process. The purpose of this study was to determine the effects of instruction method on the level of learner dissonance and how this impacts students' perceptions of learning statistics. Initial findings from data collected from a traditional teacher-centered course and from a student-centered course are presented.

As technology has been developed to carry out statistical procedures, the focus of statistics education has shifted from a procedural knowledge of statistics to a conceptual knowledge of statistics. According to the Guidelines for Assessment and Instruction in Statistics Education (GAISE) report (2005), "Achieving this knowledge will require learning some statistical techniques, but the specific techniques are not as important as the knowledge that comes from going through the process of learning them" (p. 11). Additionally, since statistics is widely used in the media and in all disciplines of research, it is important that introductory statistics classes produce critical consumers of statistics. Not only is it important that students be able to draw upon knowledge that they have learned in class, but they must also be able to use those skills they have learned and apply them to situations that may be unfamiliar.

In order to examine how the method of instruction affects students' attitudes towards statistics, perceptions of statistics, and perceptions of knowledge of statistics, two sections of undergraduate introductory statistics were taught using different instructional methods. One was taught using a traditional teacher-centered lecture approach, where students expect to be shown the process for solving problems and then mimic the procedures. The other section was taught using a student-centered non-lecture problem-solving approach (Wheatley & Abshire, 2002; Van de Walle, 2004), in which students use resources to solve the problems without first being given any formal instruction. In this paper, we examine the students' initial reactions to the course and beliefs on what they learned in the first five weeks of class. Even at this early point in the semester, valuable information can be gained from students' reflections.

Theoretical Lenses

This study explores the idea that students need to experience some amount of dissonance in order to construct meaningful knowledge. Piaget (1972), Vygotsky (1978), and Dewey (1938) have all explored the idea of dissonance as a vital element in the process of constructing knowledge. Piaget described the need for disequilibrium—a certain amount of interference with cognitive processes—as a necessary initial step in the progression of contemplating new ideas. However, too much disequilibrium can cause a hindrance in learning. Vygotsky explains that in order to learn, we must give students dissonance that is within the confines of their "zone of proximal development" or meet them where they are, based on what they know. The learner can then construct new knowledge and reach a state of re-equilibrium (Piaget), in which the uncomfortable nature of the new information is made sense of and becomes comfortable. When the discomfort is too great and outside the student's "zone of proximal development," experiences become miseducative (Dewey), and learners are no longer able to engage in the process of forming new ideas but rather shut down the process of knowledge construction altogether.

Scholars such as Houser (2006) have explored the idea that traditional teaching and learning methods do not perturb students in the way that is necessary for intellectual stimulation and growth. In the case of the lecture-based classroom, knowledge is viewed as external to the learner, ready to be handed down from the expert to the novice. Although students often seem to follow a lecture and believe that they understand and can apply the material from the lecture, the material is thought about and developed by the instructor beforehand and appears to be ready-made, easily found, and almost magically appearing (Young, 2002). The student does not experience the dissonance necessary to understand the mathematics but rather experiences a lecture in which it appears no discomfort was encountered in the formation of the mathematics. However, this type of instruction often reveals that students do not have a meaningful understanding of the material and are unable to apply the knowledge, causing them to fall back on mimicking the teachers' procedures rather than develop their own mathematical problem-solving abilities (Young).

This paper addresses how dissonance emerged early in the semester as an important element in the process of forming conceptual knowledge the courses explored in this study. Heibert and Lefevre (1986) have described conceptual knowledge as constructing an understanding of a

mathematical topic or concept that answers the underlying question: why? It is not only having the ability to do mathematics, but also having a deeper understanding of the relationships between and among mathematical concepts and topics. Further, this paper investigates how the seeming lack of this type of dissonance in a lecture-based course emerged as having contributed to only a procedural knowledge. Heibert and Lefevre (1986) have expressed procedural knowledge as a superficial understanding of a mathematical concept where an individual can go through the motions of working a problem with a reliance on algorithms and memorization but not understanding why it is done in the way it is or what its relationship to other mathematical topics is. Although procedural knowledge can indicate a breadth of ideas, it often lacks the depth. The apparent differences in the levels of dissonance were tied to the formation of procedural versus conceptual knowledge. This idea stemmed from investigating the effects of exposure to the same statistics concepts with two comparable groups of students, one in a lecture setting and another in a problem-centered learning environment.

Methodology

For this study, one section of an introductory statistics course was taught using a traditional lecture-based approach, and a second section was taught using a more student-centered problem-solving approach (Wheatley & Abshire, 2002; Van de Walle, 2004). This is a 100-level course that contains students from various majors across the university, which fulfills the statistics requirement for entry into many professional programs, such as pre-pharmacy, pre-nursing, pre-veterinary, etc. In addition, many middle grades mathematics majors (students who plan to teach mathematics in grades 5-9) choose to take this course to fulfill the statistics requirement for their major. Both sections were taught three days a week by the same instructor. The instructor holds a Ph.D. in Statistics and has taught introductory statistics at the collegiate level for over eight years. Prior to the semester of study, the instructor always taught the class using the traditional lecture-based approach. The instructor has consistently received favorable teaching evaluations, with students commenting on her relatability, organization, and effectiveness.

For the more traditional lecture-based class, the instructor provided the students with guided notes that were filled in during class. Word problems and some content were provided in the printed version of the notes, and students filled in the remaining material and solutions during class. Typically, the instructor first presented the new material and explained the process for completing certain types of problems. This was followed by an example solved by the

instructor, after which students were often presented with another example to try at their desks. One way in which the instructor attempted to engage the students was through the use of real-world examples and scenarios. Although the class was almost extensively lecture-based, five in-class activities were used during the semester to give students more practice with the material. These activities were always completed after the instructor had already lectured on the topic that the activity was designed to cover. Additionally, students turned in homework assignments approximately once a week, and these assignments were graded for correctness.

For the nontraditional class, the instructor never lectured. Instead, students were required to read the textbook and complete "homework journals" prior to coming to class. These homework journals consisted of a set of questions designed to guide the students through the material. Both conceptual and computational questions were included. Although most material could be found in the textbook, students were also encouraged to seek out other sources, such as those that can be found online. Upon coming to class, students met in small randomly-assigned groups of four to five students to discuss the homework journals. Students were told that the goal of this group work was not merely completion of the homework journal; they were to help each other with the material and share their solutions and methodology with their group. Once groups finished, the instructor led the class in a large-group discussion. Often times, students were asked to go to the board to explain a problem or concept. Even though the instructor was the leader of these discussions, much of the discussing was done by the students. Although this format of teaching was new to the instructor, she did receive guidance and input from mathematics education faculty who have taught using similar methods in their courses.

In lieu of turning in homework once a week for assessment (as done by the lecture class), students in the non-lecture class were given an "in-class journal" at the start of each class. These five-to-ten-minute in-class journals varied from closed-notes journals covering topics discussed in class to open-notes journals on material that had not yet been discussed as an entire class. All in-class journals were graded for correctness. However, students were allowed to redo them each once if they wanted to improve their grade.

Prior to the first exam, data were collected in the form of student reflections on what they had learned from the class at that point. In the class immediately preceding the first exam, both classes were asked to reflect on what they had learned as a result of being in this class. Directions stated that the students would receive full credit as long as they answered honestly.

The assignment simply stated: "What have you learned as a result of being in this class? (Note that this does not necessarily have to be statistics-related.)" Verbally, students were told that they could write about anything, from the format of the course to the material that had been covered. In each class, twenty-seven students turned in the assignment. In order to assess content knowledge at that point, both sections were given a similar (although not identical) exam, which tested the topics of graphical and numerical descriptive statistics, correlation, and simple linear regression.

An interpretive framework was used to analyze the data collected through the student reflections (LeCompte & Schensul, 1999). In order for the researchers to develop an overall idea of the students' beliefs about what they had learned, the student responses were read multiple times, both individually by the researchers and collectively as a research team. The researchers consisted of a statistics professor (who was also the course instructor) and two mathematics education professors. Each researcher evaluated and categorized each response individually. Then the researchers met as a group to compare individual categorizations. Any deviations were discussed. The researchers noticed several themes and categorized each response into one or more themes. The researchers also sought to find answers to the following questions regarding the lecture vs. non-lecture students: (1) Are students displaying dissonance? (2) Are students displaying a procedural or a conceptual understanding of statistics? Finally, quotes from student responses were extracted to corroborate the findings.

Findings

Analysis of the student reflections exposed major findings about the differences between the lecture and non-lecture classes. The major themes are as follows.

Dissonance and Learning Style

Based on the lack of lecture-student comments regarding the structure of the class, we inferred that those students were either comfortable with the traditional style of teaching or uncomfortable commenting openly on the course. Only 6 of the 27 lecture students even commented on the instructional format of the class. Of these six, only two had negative comments; both of these students expressed concern with what they believed to be the fast pace of the class. One student said, "This class is moving very fast for me and I feel really overwhelmed at times. I just wish we could slow down just a little bit so I can really grasp the concepts more fully and in depth." The four positive comments indicated that students liked

examples, the guided notes, and the overall structure of the course. One student mentioned, "I really like the set up of the class, and how lecture & HW & upcomming [sic] tests are very well gone over & I know what is expected of me as a student in this class." Another student said, "I also like the way we take notes. I like having part of it wrote out and then also having to fill in other parts, it keeps the class involved and doesn't make you feel like you are going to write your hand off."

On the other hand, nearly 75% of the non-lecture students commented on the teaching method that was utilized in the class. These comments ranged from complete discontent with the instructional format of the class to complete satisfaction. Some comments indicated that students saw benefits with the instructional method, but they still were not sure that they liked it. It appeared that the nontraditional teaching style caused dissonance for many students, but this seemed to be a good dissonance. Several students made explicit statements that they would prefer lecture because they believed that they would be able to better understand the content. One student said, "Being in groups, trying to help each other learn, is not at all helpful to me. I am not mathematically inclined as is and being in these groups and being expected to know everything without generic structured, lecture style teaching is hard for me." Although some students saw the benefits of student-centered learning, some claimed that they would still prefer having lecture. "[E]ven though the journals are helpful it gives me the impression that I'm teaching myself and I like the traditional math class."

Although this format of teaching was new to almost all students in the non-lecture class and uncomfortable for many, several students began to see the benefits. One student noted, "The learning style in this class is much different than any class that I've had. However, it seems to be working so far." Another student noticed the benefits of striving for conceptual understanding as she progressed through the material as opposed to mindlessly taking notes from a lecture and studying them before a test: "I have learned that while it's not easy teaching yourself the material it is beneficial. Being in groups is helpful, because most of the time if I didn't know an answer one of my group members [did]. I've learned that being in a class like this, it seems almost as if I don't have to study as much, because I've studied throughout to do the homework journals." Even students who had previously taken a statistics course noticed the benefits of this nontraditional format. For example, one student said, "I've taken stats before so most of what we have covered has been review for me but I understand it better this time. Before I just

memorized the formulas and could get the right answer but I didn't really know what the answer meant."

Superficial/Meaningful Knowledge

Since students were asked what they had learned as a result of being in this class, most students in the lecture class took this question at a surface level. Sixty percent of students in the lecture class responded with a list of statistical topics that they believe they had learned as a result of being in this course. All content-related comments were limited to explicit statistical topics that were addressed within the confines of the class (statistical terms, computer programs, etc.). This knowledge seemed to be procedural as opposed to conceptual. This is exemplified in student reflections that addressed the statistical terms they learned as opposed to indicating any true understanding of how to apply them. Comments like "I've learned alot about using statistics to find the mean, median & modes of data" and "I have learned how to make a box plot" were quite common. Students tended to address the "how" and "what" they have learned, but not the "why" behind it. Although comfortable with teaching style, the information seemed to be an inch deep and a mile wide for some students. One student said, "I'm not great at it, and I really couldn't tell you when to use anything we've been taught, but I get it at least. I understand (I think, anyway!)" The student then went on to list various statistical terms that had been presented in class.

Twenty percent of students in the lecture class said that they found statistics to be a difficult field of study, saying things such as "… it's actually more complex than I thought it would be." Although some students in the non-lecture class mentioned discomfort with the class, all of those comments were directed towards the style of instruction, not the material itself.

In the non-lecture section, comments moved beyond the confines of explicit topics discussed in class. Although fewer than 25% of students in the non-lecture class mentioned statistical terms in their response, 30% discussed meaningful knowledge that moved beyond statistical concepts. As an example, students mentioned applying their statistical knowledge to other courses. One student said, "So far I have learned how to calculate/interpret data that is useful in my own research." Another student mentioned that he had learned "problem solving from a mental standpoint not just math." Other students noticed the benefit of being actively involved in one's own learning and saw how they could apply this to other classes, even when the course is taught in a more traditional manner. One of the reflections stated, "Being in this class has

taught me that I truly learn best through verbal explanation. I retain more information if I am able to verbally explain a concept to other people. Although I am able to pick up information while working alone, the information 'sticks' better if I can talk about it. I find myself doing this in other classes besides statistics now, and it really helps!"

The reflections indicated an unexpected benefit of this teaching method, an improvement in interpersonal skills. Students are often unwilling to or are uncomfortable with participating in a mathematics course. The nature of the non-lecture course fosters the students' willingness to communicate with each other by requiring small-group and whole-group discussions about the statistics content. In turn, some students realized how the format of this class was boosting their confidence, which is indicated by the following response: "This class has given me the opportunity to be open and give my opinion which to be honest I didn't know I was capable of." Even someone who believes they are not good with other people sees how this method of instruction can improve that: "I do enjoy the group work, however I'm not a very good people person…I guess this will help me deal with that." This is especially noteworthy because no students in the lecture section made any mention of interpersonal skills or their abilities to interact with other people.

Confusing Learning with Doing

Some reflections from the non-lecture class indicated that students equate learning with knowing procedure. Students feel that if they can mimic a process and get the correct answer, then they have learned and understood the material. The comments regarding this issue were taken from the non-lecture section where dissonance had been created by asking students to conceptually understand statistical concepts without the instructor simply writing definitions and examples on the board. A student commented, "I haven't learned much of anything in this class because I learn better from lectures and being shown step by step, rather than trying to learn from a group when at least three out five all have different answers." Yet another student said, "I know that I learn and remember information much better if the professor lectures and shows me how to correctly solve the problem." Even a student who likes working in groups still believed that she was not learning as much as she would from a lecture, stating "I have learned that I learn better in a lecture based class, but enjoy a discussion/group based class more." It is as though she is equating learning with lack of enjoyment. It is interesting to note that there were no comments from the lecture class that indicated that they would prefer or learn better from a

different instructional technique.

Although many students in the non-lecture class indicated a preference for lecture and/or a lack of comfort with this new style, they outperformed the lecture-based section on the first exam (given just one class period after they were asked to reflect on what they had learned). Based on the scores on the first exam, the non-lecture class showed a deeper understanding of the material. The mean grade for the non-lecture class was over six points higher than that of the lecture class, while the median was 5.5 points higher. Table 1 shows the distribution of letter grades for both sections.

Table 1.

Distribution of grades

	A	B	C	D	F
Lecture	27.59%	34.48%	17.24%	6.90%	13.79%
Non-Lecture	48.15%	29.63%	7.41%	3.70%	11.11%

Implications

Research regarding teaching methods often looks at the end result of the study, seeking to assess how the method has affected student learning or attitudes about the subject area. We believe that student feelings throughout the semester, and not just at the end, are important. By understanding student beliefs and reactions at the beginning of the semester, it is possible to address them at that point. Since many students in the non-lecture class were experiencing dissonance just five weeks into the semester, the instructor was able to deal with this issue. She reassured the students that these feelings of frustration and discomfort were normal and to be expected and that any student feeling this way was not alone. She told the students that the purpose of completing the homework journals, without having had explicit guidance or lecture-based instruction, was not to simply get the correct answer. The purpose, rather, was to develop greater problem solving and collaboration skills.

Furthermore, while the students in the non-lecture class were experiencing some frustrations and discomfort, they were not alone; the instructor also experienced some of these feelings. This teaching method was new to the instructor, who was quite comfortable with a lecture-style class. It was difficult for the teacher to learn to take a step back and not intervene when the group

discussions led to uncertainty and further questions. However, as the semester progressed, the instructor's anxiety decreased greatly.

Although the lecture students seemed more comfortable with the style of the class, this did not necessarily lead to greater student learning (as measured by performance on the first exam). In future papers, the researchers hope to investigate how students' beliefs on what they learned changed, if at all, throughout the course of the semester. We will also investigate how students' attitudes about the field of statistics were affected by this course.

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APPLYING CONCEPTUAL METAPHOR THEORY TO UNDERSTAND MATHEMATICAL PROBLEM SOLVING

Sean Yee
Kent State University
Syee1@kent.edu

My pilot study demonstrated significant results in how students perceive mathematical problem solving using conceptual metaphor theory (CMT). Currently, a larger study is underway to confirm these results and compare them to their teacher's perceptions of mathematical problem solving. Problem solving has been perceived for the last 30 years as a process primarily due to the work of cognitive science. However, my pilot study showed that many students do not perceive problem solving as a process, rather metaphors such as journeys, buildings, or discovery. My current study expands on the pilot study's results to help teachers and students understand each other's perception of problem solving.

For the last 60 years, mathematical problem solving has been a prevalent dimension of mathematics education. Mathematics educators have perceived mathematical problem solving as a heuristic process (Pólya, 1945), a logic-based program (Newell & Simon, 1972), a means of inductive and deductive discovery (Lakatos, 1976, 1999), a framework with multiple dimensions (Schoenfeld, 1985), methodologies with multiple variables (Kilpatrick, 2004), a standard (NCTM, 1989), and a model-eliciting activity (Lesh & Zawojewski, 2007). These varying perspectives are difficult for educators to agree upon, but even worse for students, who are told that problem solving is vital for their mathematical education. The goal of my study is to approach problem solving from the perspective of the party that has the most to gain or lose from its interpretation, the student. It is vital to understand how students perceive problem solving if educators are to discuss mathematical problem solving in a meaningful manner with the student.

Theoretical Framework

Cognitive Science and Modeling

Many pioneers of cognitive science attempted to model student's problem solving ability as a *process*. The metaphor of *process* originated from Newell's (1972) inspiration of solving problems with computers. Indeed, thinking of problem solving as a mechanical operation aligns directly with the artificial intelligence aspect of cognitive science. Early cognitive science research followed methodical techniques. Such theories were derived from observing students solving problems, reflecting on the techniques of problem solving and prior research, and creating a process that modeled how the student solved problems or aspects of problems (Kilpatrick, 2004; Newell & Simon, 1972). Theories were tested and data corroborated their

design. However, these theoretical models were consistently perceived, interpreted, and developed by the researcher. The lens of the researcher was the primary lens for the model's development, not the student's lens.

Lesh and Doerr (2003) offer a fresh perspective that supports the need to deviate from the notion of a problem-solving *process* to the notion of problem solving as a model-eliciting activity. This allows Lesh and Doerr to explicitly observe the "how and why" of problem solving by suggesting that the student should model how they solved the problem. Specifically, Lesh and Doerr argue for student-generated models of the problems where the solutions are part of the model, not the end result of a process. Ontologically, Lesh and Doerr deconstruct the traditional dichotic view of understanding (i.e. you either get it or you don't) and replace it with a cycles of modeling within a model-eliciting activity. These cycles allow the teacher to observe how the student modifies their model of understanding of a problem so that the teacher may understand how the student perceives the problem.

My research was inspired by Schoenfeld's (1992) framework and Lesh and Doerr's (2003) modeling design. I focus on letting students model their perception of problem solving, instead of the researcher, to minimize misinterpretations by the researcher. However there must be some structure to observe the student's model of problem solving. We require a design that will minimize misinterpretation and emphasize the student's ability to communicate their thoughts. For this reason, linguistics plays a vital interdisciplinary role. Specifically, I will use the linguistic structure of metaphors to minimize the researcher's involvement in modeling because metaphors allow the student to generate an experiential mapping through conceptual metaphor theory so as to improve their explanation and perception of problem solving. Thus the question driving my research is: what do the metaphors used by students tell educators about the student's perception of mathematical problem solving? My hypothesis is that high school students express a plethora of conceptual metaphors. These conceptual metaphors are relevant to educators and researchers attempting to understand and communicate perceptions of mathematical problem solving to students.

Conceptual Metaphor Theory

Metaphors are not only for poetic representations, but for transfer of ideas. New knowledge is attained by relating (not isomorphically, but analogously) aspects of old knowledge from multiple individually perceived experiences. Indeed, Presmeg (1997) reminds us that the Greek

word, *metaphora*, means to transfer or carry over. Traditionally, a metaphor literally denotes one figure of speech as another figure of speech (Merriam-Webster, 2011). Yet the influence of this transfer from a cognitive perspective encouraged Lakoff and Johnson (2003) to expand on the conduit metaphor to create the conceptual metaphor. In the conceptual metaphor, the literal relationship between figures of speech is replaced with a conceptual mapping between linguistic expressions (Lakoff, 1993). For example, the literal metaphor "Your theoretical framework lacks a solid foundation" would have the conceptual metaphor, THEORIES ARE BUILDINGS, attached to it. Thus with each metaphor, you have two parts:

- Literal Metaphor - The actual literal expression.
- Conceptual Metaphor - TARGET DOMAIN IS SOURCE DOMAIN

In all conceptual metaphors, there is a source domain and a target domain. The source domain is the experientially-known domain and the related concept is the target domain. Thus in the metaphorical linguistic expression "The solution escapes me", the target domain is solutions while the source domain is prey. Hence the conceptual metaphor is SOLUTIONS ARE PREY. This demonstrates the superordinate conceptual metaphor (Kövecses , 2006) PROBLEM SOLVING IS HUNTING. It is important to note that despite the use of the being verb, "IS", the phrase is unidirectional (TARGET DOMAIN → SOURCE DOMAIN).

Methodology

Pilot Study

My pilot study explored how metaphors were used by high school students while solving a set of three mathematical problems. I had nine voluntary suburban high school participants ranging from 9th to 11th grade. These students were given three mathematical problems and 30 minutes to attempt to solve and justify as many questions as they could. The students were video-recorded and after the initial 30 minutes, the students watched the video of him/herself solving the problems and were instructed to explain their thinking and problem-solving techniques. Each problem varied in its use of deductive and inductive reasoning. Problems were chosen so that little prior knowledge was necessary. Additionally, the metaphors in the three problems were limited so as to evoke the student's metaphors.

Results from the pilot study suggest that the student's use of metaphors for problem solving were complex yet coherent. Specifically, two results encouraged this current study. First, students modeled their metacognitive problem solving techniques clearly and concisely. For

example, students were aware of their subconscious. Multiple students referred to the need to "give their subconscious time to work on the problem". Other students said their mind was "playing tricks on them". In both situations, aspects of their mind were seen as separate from themselves generating the conceptual metaphor THE MIND IS A SEPARATE PERSON. This encouraged trust that high school students could ontologically perceive their problem-solving process. Specifically, high school students were capable of interpreting their problem solving skills, and articulate them appropriately.

Secondly, the use of conceptual metaphor theory (CMT; Danesi, 2007; Lakoff & Nunez 2000) offered a significantly rich perception of problem solving. Danesi (2007) applied CMT analysis to assist in eighth graders' and teachers' understanding of word problems. Specifically, abstract metaphorical ideas, such as the belief that numbers lie on a *line*, were interpreted through the student's lens using *concretization* (The act of making an intangible idea concrete through similarity or comparison). In my pilot study, CMT analysis revealed students perceive problem solving through a variety of experiences yet all nine students consistently referred to problem solving using four conceptual metaphors; mathematical problem solving as a journey, a building, a discovery, or an experiment. The results are demonstrated on Table 1 below.

Table 1.

Pilot Study Results using CMT Analysis of Problem Solving

TARGET DOMAIN	→	SOURCE DOMAIN	Percentage of Participants
PROBLEM SOLVING	IS	BUILDING	100%
		A JOURNEY	100%
		DISCOVERING	67%
		EXPERIMENTING	56%
		A MACHINE	56%
		PLAYING	44%
		SEARCHING	44%
		TRICKS	33%
		STRATEGIES	22%
		A PRODUCT	22%
		A DESTINATION	22%
		A GOAL	22%
		ILLUMINATING	11%
		CHOOSING	11%
		TRAVELING	11%
		APPROXIMATING	11%
		A HUNT	11%

The percentages show the percentage of students who used the conceptual metaphor at least once in the interview. These conceptual metaphors were coherent due to the fluid transition of the student from one conceptual metaphor to the next within the same mathematical problem. Many conclusions can be drawn from this data. For example, despite our emphasis in current mathematics education on cognitive understanding of heuristics as a mathematical *process* involved in solving problems, student's only claimed it to be a process (machine) 56% of the time. Perhaps other more student-relevant metaphors such as JOURNEYS should be studied by mathematics educators and researchers in hopes to integrate them into current curricula. Many students used literal metaphors such as, "I don't know where to begin" or "I'm lost, I don't know where to go". Other results relative to CMT analysis were also discovered, but omitted from this brief proceeding.

Three limitations occurred in the pilot study that suggested a need for the current study to validate the results of the pilot study. First, students were given problems chosen by the researcher in the pilot study. The questions should not be provoked by the researcher to have significance to students, but rather by the teacher. Second, the sample size limited the quantitative value as there were only nine participants. A sample size of over 20 participants is necessary to have statistically significant results (Central Limit Theorem). Third, the pilot study focused on a population of students with immense mathematical proclivity (specific criteria were used), rather than a more generalizable audience. The current study will still focus on students who have a mathematical inclination (honors students), but no other criteria beyond this will be used to initially determine participants. It is important to note that the data of the current study is still being analyzed and only some initial results are discussed.

Current Study

The expectations for the current study are a rich set of literal and conceptual metaphors from teachers and students describing mathematical problem solving. Thus my hypothesis is that the students' set of conceptual metaphors for mathematical problem solving will be greater than the current metaphors used in research. Additionally, this study will illustrate similarities and differences in the student's and teacher's source domains relative to the target domain of problem solving. The participants for the current study are 22 suburban high school students and their two honors geometry teachers. Geometry was chosen due to the value of proofs to problem solving. Specifically, proofs encourage students to justify their conclusions and thus lead to

perceptions of the student about problem solving (Lesh & Zawojewski, 2007). Within the chosen high school, two teachers were responsible for teaching all six of the honors geometry classes. Both participating teachers agreed upon a common assessment at the end of each chapter of the chosen text (College Preparatory Mathematics, Geometry). The researcher chose one problem on the common assessment that requires problem solving according to Lesh and Zawojewski's (2007) definition. The teachers interviewed individually with the researcher via video prior to giving the assessment so as to explain what problem solving techniques they expect the students to use. The semi-structured interview with the teacher was limited to 10-15 minutes. The interviews were transcribed and literal metaphors corresponding to conceptual metaphors were recorded.

After students took the common assessment and the assessments were graded by their teacher, students who had volunteered to be part of the study were chosen by the researcher by a partitioned randomization demonstrated in Table 2 below.

Table 2.

Partitioned Randomization Participation for Each Assessment

Student-Teacher Alignment	Teacher ALPHA	Teacher BETA
Students whose solution aligned well to the teachers expectations (67-100%)	1-2 Students	1-2 Students
Students whose solution aligned moderately to the teachers expectations (34-67%)	1-2 Students	1-2 Students
Students whose solution aligned poorly to the teachers expectations (0-34%)	1-2 Students	1-2 Students
TOTAL per assessment	3-6 Students	3-6 Students

The students whose assessments were chosen randomly by the researcher were interviewed (semi-structured) by video for 10-15 minutes. This was done for the first three common assessments for the honors geometry course including a total of 22 student interviews.

Findings and Conclusions

The current study's data analysis is in progress and thus only preliminary results are available. One clearly significant result supports the findings of the pilot study. In the current

study, only 45% of the students used the conceptual metaphor of PROBLEM SOLVING IS A PROCESS. Moreover, literal metaphors such as "I couldn't see how to solve the problem" and "it wasn't completely obvious to me" suggest a new conceptual metaphor of PROBLEM SOLVING IS VISUALIZING. This buttresses the pilot study's conclusion that student's perceptions of mathematical problem solving are not limited to processes and procedural understanding.

These results open an entire new direction for problem-solving research with the use of CMT analysis. Usually, if one wants to abbreviate results into a succinct description, numerical methods such as quantitative analysis are applied. CMT analysis offers a qualitative method to summarize experiential perceptions. In my current study, problem solving is the topic of interest. Yet this methodology (CMT analysis) searches for the founding experiences of the students or teacher and associates them to the given source domain. This is a qualitative methodology deriving explicit language from participant's experiences.

The contribution of my studies and CMT analysis to mathematics education is to identify the conceptual metaphors, the embodied cognitive experiences, of the student and relate them to the teacher's conceptual metaphors. Problem solving is a complex and important issue in mathematics education, and if the two main parties (teachers and students) cannot find common ground from which to discuss such topics, education and assessment of those topics will suffer. This research directly speaks to the tenets of RCML to stimulate, generate, and disseminate research to aid in the learning of mathematics. Moreover, teacher education can benefit from CMT analysis because having firm ground for the language of mathematical problem solving allows teachers to improve student learning directly.

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WHAT'S THE MERiT IN RESEARCH COLLABORATION?

Mary B. Swarthout
Sam Houston State University
swarthout@shsu.edu

Winifred A. Mallam
Texas Woman's University
WMallam@mail.twu.edu

This paper shares the beginnings and impact of the MERiT (Mathematics Education Research in Texas) Conference on the mathematics education research community in North, East and South Texas. MERiT participants share research expertise and results, collaborate on new and continuing research projects, coordinate research agendas, and disseminate work to state, regional, and national audiences interested in mathematics learning. The MERiT approach to professional collaboration holds promise as a tool to encourage continued research by faculty at small to medium-sized institutions, particularly those who are the lone mathematics education faculty member and may need support for an active research life.

The shortage in the supply of graduates with doctoral degrees in mathematics education has been a topic for discussion over the past several years. A status report by Reys, Glasgow, Teuscher, and Nevels (2007) documented the shortfall and discussed the need to engage in discussion and action to improve doctoral programs in the discipline. A connected area of concern focuses on the need to support faculty – those just entering the profession as well as more seasoned members, ensuring that all flourish in higher education – and are able to produce the level of research needed to receive tenure, promotion and contribute to the research base. How can the cadre of mathematics teacher educators encourage and actively engage with one another in ways that provide opportunities and avenues for collaborative research? Are there models that could be used to create beneficial connections for faculty members within the mathematics education community?

The purpose of this paper is to share the development and impact of the MERiT (Mathematics Education Research in Texas) Conference on the mathematics education research community in North, East and South Texas using the literature of collaboration to establish the importance of such an endeavor.

Literature Review on Research Collaboration

Collaboratives form when individuals share a vision and communicate with each other as they aim to fulfill their vision (Leonard & Leonard, 2002).

Characteristics of Models

The importance of collaboration has been promoted by professional organizations for accreditation and teacher preparation and has been included as part of their standards or

goal/mission. Ross et al. (2005, p. 278) noted that "If collaboration is a valued skill for teachers, then it is essential that teacher educators find ways to make collaboration a more integral part of the university context". Likewise collaboration has been promoted by associations such as the Association of Teacher Educators (ATE), the National Association for the Accreditation of Teacher Education (NCATE), Interstate New Teacher Assessment and Support Consortium (INTASC), and the National Board for Professional Teaching Standards (NBPTS). They all incorporate collaboration in their standards. The Association of Mathematics Teacher Educators (AMTE) also includes collaboration in their mission and goals.

Members of the collaborative should have a shared vision and communicate with each other as they aim to fulfill their goal (Leonard & Leonard, 2002). The term professional learning community (PLC) refers to a group of teachers, usually within a school, who work cooperatively to improve student learning (Cory et al., 2010). It generally refers to the K-12 level but the concept can be applied to institutions of higher learning. For the PLC to be successful, Hord (2009) identified several conditions. He suggested a dedicated time and space for meetings, including rotation of meeting venues. Face-to-face meetings are preferred as Parr and Ward (2006) found that for K-12 collaborations interacting via the internet were not as productive.

Collaboratives can be formed at the university, 2-year college, and K-12 levels or across K-16. MERiT is an example of a university collaborative comprised of mathematics educators with a common goal. At the 2-year college, Minkler (2002) used the term learning communities (LC) to refer to building a community of learners among students and faculty. Andrews and Lewis (2004) discussed PLCs amongst K-12 schools and noted that teacher leaders emerge as a result of participation in PLCs. PLCs in Chinese elementary schools were formed based on institutional support, leadership of the principal, and the teachers' own initiative (Sargent and Hannum, 2009). Frost, Coomes, and Lindeblad (2009) researched the characteristics and outcomes of a collaborative professional development project between secondary school mathematics teachers and postsecondary mathematics faculty. Their focus was on the difficulties that students face when they transition from secondary mathematics courses to postsecondary mathematics courses.

Benefits

Fostering faculty connections, whether establishing ties between different universities or even between different disciplines on the same campus, can have a positive impact on the

research of participating faculty. Akerson, Medina, and Wang (2002) found that an assistant professor of environmental engineering who collaborated with science education and higher education faculty members benefited from peer faculty support. The areas of gain included questioning strategies, thinking time for students, increased students' participation, and implementation of student-designed field research projects.

Partnerships help to lessen the feelings of isolation that faculty, especially new faculty, feel (Savage, Karp, & Logue, 2004). When participating in research partnerships that reflect on teaching, new faculty recognize the complementary nature of research and teaching (Akerson, Medina & Wang, 2002).

MERiT Model

As with any field, the professional development needs of mathematics education researchers are a unique challenge to fulfill. There may be one or only a handful of such researchers in a Mathematics Department or College of Education. This leaves the faculty navigating the way through these challenges in their isolated situation. Forming collaboratives helps to fill this void.

The initial idea for a conference to organize mathematics education faculty from a regional area in Texas began with the faculty at Sam Houston State University (SHSU), a regional state university in East Texas. With eight mathematics teacher educators within the Department of Mathematics and Statistics, all holding doctorates in mathematics education, there is a critical mass of faculty, an important component to the healthy research activity of the individual members. The SHSU faculty group, using a research seminar weekly to share, read, react, and support each other in projects and remaining current in the discipline, wondered what faculty in other situations did to support their research work. The question arose, How do others maintain an active research agenda, especially if they are the lone faculty member at their college or university? With support from the department, the college dean, the graduate dean, and the provost, the SHSU faculty decided to invite faculty in the region to participate in an event to begin a conversation about what was needed to be vibrant and active researchers and how we could work collaboratively on research projects – to serve faculty needs and contribute quality research to the knowledge base of the discipline. The goal from the start in 2008 was to allow participants to share research interests and current work, promote collaboration across campuses, and provide informal mentoring opportunities for newer researchers and faculty members.

The structure of each MERiT meeting is simple – but has been viewed by the participants as an interaction was well worth their time. This is indicated by a 100% response rate of Agree or Strongly Agree that attending the conference was worth their time investment, based on a conference survey completed by participants in 2010 and 2011. The conference encompasses less than 24 hours of time, which begins on Thursday evening with a dinner and an opening speaker. Friday morning is used to ensure that participants get to know one another. It also provides an opportunity for participants to work in collaborative groups on current projects, work to form new research groups, get feedback on manuscripts, and seek help in data analysis. Interactions have maintained an informal feel to address the needs of the group attending. The conference, held each fall semester, has an average of 20 to 25 participants from North, East, and South Texas institutions of higher education, both private and public. Serving as host, the mathematics education faculty at SHSU, with monetary support from the department and graduate school, covers the meal costs and the cost of hotel rooms for Thursday evening. Within this framework, MERiT participants have established connections that contribute to professional development and growth as researchers in the discipline.

Collaboratives formed and their research accomplishments

A significant outcome of the MERiT Conference model has been the establishment of a number of both formal and informal collaboratives. This section will discuss two formal, organized collaborative born from the MERiT experience (Mathematics Education Research Group in North Texas (MERGiNT) and Algebra Teacher Self-Efficacy Instrument (ATSI) Collaborative), and provide several examples of the informal collaborations that have developed over the past five years.

The MERGiNT is comprised of nine member institutions representing four research and teaching universities. The members are at various stages of their academic careers, ranging in rank from assistant professors to full professors. Only two of the participating institutions have more than two faculty members in mathematics education. The group was formed to address the sense of isolation that results in the need for support and mentoring from colleagues in the same field. Members meet monthly, rotating from campus-to-campus. Activities include peer review of members' draft papers that they intend to publish or present at a professional meeting. As a team, MERGiNT has presented papers at research conferences and submitted papers for publication in professional journals.

The ATSI Collaborative consists of ten faculty members representing nine Texas universities. With a combination of face-to-face meetings and connecting through technology (conference calls, Skype), members have collaborated with the aim of developing and validating the ATSI Instrument. Similar to Parr and Wood's (2006) findings, based on anecdotal evidence, members of the ATSI Collaborative found that meeting face-to-face is more productive. As a team, members of the ATSI Collaborative have presented the development of the instrument at professional meetings and have submitted proposals to seek funds to support their travel as they refine, validate, and market the instrument.

Along with formal collaboratives, the participants in MERiT have found informal ways to connect and support research work. For example, the pairing of a faculty member with data collected from one institution with a faculty member at another university with extensive experience with statistical analysis, resulted in a publication in a peer-reviewed journal. Another example is the identification of fellow mathematics educators that can serve grant projects as outside evaluators or that can serve on advisory boards for projects. These informal connections continue to bear fruit through publications, presentations, and the design of future research agendas and ventures in mathematics education research.

Conclusion

Faculty in mathematics education will continue to need professional support from one another – whether located on the same campus or within the same regional area. The suggested MERiT model represents one such avenue to organize support for research activity. Providing venues for faculty to talk, discuss, and identify areas of commonality or intersection is necessary to foster the development of collaborative communities that serve both individual faculty members and the discipline at large through solid research activities that grow the base of knowledge about mathematics teaching and learning at all levels.

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